'What to say in front of the blackboard'
- a brief tutorial.

a).

$$\lim_{x \to 0} \frac{\sin 2x + \sin 3x}{\sin 4x + \sin 5x} =$$

→ Sine of two x plus sine of three x all over sine of four x plus sine of five x.

$$= \lim_{x \to 0} \frac{\sin 2x}{\sin 4x + \sin 5x} + \frac{\sin 3x}{\sin 4x + \sin 5x} =$$

→ First, I split the fraction into two parts, in order to have only one element in numerator.

$$= \lim_{x \to 0} \frac{1}{\frac{\sin 4x + \sin 5x}{\sin 2x}} + \frac{1}{\frac{\sin 4x + \sin 5x}{\sin 3x}} =$$

→ Then I divide every part of the fraction by the reverse of numerator.

$$= \lim_{x \to 0} \frac{1}{\frac{\sin 4x}{\sin 2x} + \frac{\sin 5x}{\sin 2x}} + \frac{1}{\frac{\sin 4x}{\sin 3x} + \frac{\sin 5x}{\sin 3x}} =$$

→ Now I also split the fraction into two parts (again to have only one element in numerator).

$$= \lim_{x \to 0} \frac{1}{\underbrace{\frac{\sin 5x}{5x}}_{\to 1} \cdot \underbrace{\frac{2x}{\sin 2x}}_{\to 1} \cdot \frac{5x}{2x} + \underbrace{\frac{\sin 4x}{4x}}_{\to 1} \cdot \frac{4x}{1} \cdot \frac{\overbrace{2x}^{\to 1}}{2x \sin 2x}} +$$

$$+ \frac{1}{\underbrace{\frac{\sin 4x}{4x}}_{\to 1} \cdot \overbrace{\frac{3x}{\sin 3x}}^{\to 1} \cdot \frac{4x}{3x} + \underbrace{\frac{\sin 5x}{5x}}_{\to 1} \cdot \overbrace{\frac{3x}{\sin 3x}}^{\to 1} \cdot \frac{5x}{3x}} =$$

→ Now I'm using a property which says that the sine of alpha over alpha tends to 1, and that's how I get rid of the sines.

$$= \frac{1}{2 + \frac{5}{2}} + \frac{1}{\frac{4}{3} + \frac{5}{3}} = \frac{1}{4{,}5} + \frac{1}{3} = \frac{1}{7{,}5} = \frac{2}{15}$$

→ And the final result will be two divided by fiveteen.

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b).

$$\lim_{x \to \infty} \left(\frac{x^2+2}{x^2-5}\right)^{x^2} =$$

The limit of x square plus two all over x square minus 5 all to the power of x square.

$$= \lim_{x \to \infty} \left(\frac{x^2-5+7}{x^2-5}\right)^{x^2} =$$

First of all, I'm going to have a form which will allow me to write the content of the brackets as 1 plus „something". So, I write two as minus five plus seven.

$$= \lim_{x \to \infty} \left(1+\frac{7}{x^2-5}\right)^{x^2} =$$

Now I can simplify x square minus 5 and I obtain one.

$$= \lim_{x \to \infty} \underbrace{\left(\left(1+\frac{7}{x^2-5}\right)^{\frac{x^2-5}{7}}\right)}_{\Rightarrow e}{}^{\overbrace{\frac{7}{x^2-5} \cdot x^2}^{=7}} =$$

In the next step I multiply and divide the power by x square minus five all over seven. Expression in orange frame tends to e and the expression in the pink frame tends to seven. So my final result is e to the power of seven.

$$= e^7$$

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