"What to say in front of the blackboard" - a brief tutorial

a)

$$\lim_{n\to\infty} \frac{\sqrt[3]{n^2+1}-1}{\sqrt[3]{n^2-n}}$$

The limit of the cubic root of n square plus 1 and minus 1, all over the cubic root of n square minus n, as n tends to infinity.

$$= \left[\frac{\infty}{\infty}\right] =$$

First of all, I notice that I cannot just substitule infinity for n, because I will get an indeterminate symbol "infinity over infinity".

$$\lim_{n\to\infty} \frac{\dfrac{\sqrt[3]{n^2+1}-1}{\sqrt[3]{n^2}}}{\dfrac{\sqrt[3]{n^2+n}}{\sqrt[3]{n^2}}}$$

In the next step, I divide both parts of the fraction by the cubic root of n square.

$$= \lim_{n\to\infty} \frac{\dfrac{\sqrt[3]{n^2+1}}{\sqrt[3]{n^2}} - \underbrace{\boxed{\dfrac{1}{\sqrt[3]{n^2}}}}_{\to 0}}{\sqrt[3]{\dfrac{n^2+n}{n^2}}}$$

And I split the numerator along the minus sign - one over the cubic root of n square tends to zero.

$$= \lim_{n\to\infty} \frac{\sqrt[3]{1+\frac{1}{n^2}}}{\sqrt[3]{1-\frac{1}{n}}} \qquad \frac{1}{n^2}\to 0, \quad \frac{1}{n}\to 0$$

In this step I simplify the content of the roots and I get the cubic root of 1 plus 1 over n square and all over the cubic root of 1 minus 1 over n. Fractions with n's tends to zero

$$= 1$$

and final result is 1.

Author: Agata Myszkowska
I year student / EPM

b)

$$\lim_{x \to 0} \frac{\sin 8x}{4x}$$

The limit of sin 8x over 4x, as x tends to 0.

$$=$$

$$\lim_{x \to 0} \frac{2 \boxed{\sin 8x}}{\boxed{8x}} \to 1$$

Now I multiply everythig by 2. I can use it to simplify the function, because the sine of some alpha over alpha tends to one as alpha tends to 0.

$$=$$

$$2$$

And in final result I obtained is 2.

c)

$$\lim_{x \to 0} \frac{\sin 7x + \sin 3x}{\sin 4x}$$

The limit of sin 7x plus sin 3x, all over sin 4x, as x tends to 0.

$$=$$

$$\lim_{x \to 0} \frac{7 \frac{\sin 7x}{7x} + 3 \frac{\sin 3x}{3x}}{4 \frac{\sin 4x}{4x}}$$

In the next step I can use the property from previous example. The labeled fragments tend to 1, so they will cancel.

$$=$$

$$\frac{7+3}{4}$$

The result will be seven plus three all over four

$$=$$

$$\frac{5}{2}$$

And final result is 5 over 2.

Author: Agata Myszkowska
I year student / EPM

d)

$$\lim_{n\to\infty}\left(\frac{n+5}{n}\right)^{3n}$$

The limit of, in the brackets n plus 5, all over n, and all this to the power of 3n, as n tends to infinity

$$\|$$

$$\lim_{n\to\infty}\left(1+\frac{5}{n}\right)^{3n}$$

First, I want to get form (1+"something") to some power, so I split the fraction along the plus sign. I received a simpler equation: in brackets 1 plus 5 over n, all to the power of 3n

$$\|$$

$$\lim_{n\to\infty}\left(1+\frac{1}{\frac{n}{5}}\right)^{3n}$$

In the next step I reverse the second component in brackets: it is 1 over n over 5

$$\|$$

$$\lim_{n\to\infty}\boxed{\left(1+\frac{1}{\frac{n}{5}}\right)^{\frac{n}{5}}}^{\cdot\frac{5}{n}\cdot 3n} \to e$$

In this step the power changed - I will multiply it and divide it by n over 5. The labeled fragment of the equation tends to e.

$$\|$$

$$\lim_{n\to\infty} e^{\frac{15n}{n}}$$

After this step, the limit takes the form e to the power of 15 n over n

$$\|$$

$$e^{15}$$

And the final result is e to the power 15.