EXERCISE: DETERMINE WHETHER THE SERIES IS DIVERGENT, ABSOLUTELY CONVERGENT OR CONDITIONALLY CONVERGENT

$$\sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{n+3}{n+5}$$

The summation, from 1 to infinity of terms: minus 1 to the power of n+1 times n plus 3 over n plus 5

$$a_n = \frac{n+3}{n+5}$$

I notice that it's an alternating series, so I will consider a_n to be only n plus 3 over n plus 5 [without the $(-1)^{n+1}$ part)

In this step, we need to perform the alternating series test. It involves checking three conditions.

1° $a_n \overset{?}{>} 0$

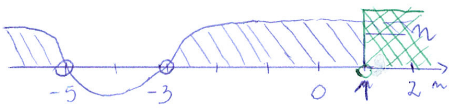
1° is a_n greater than zero for all $n \geq 1$?

$$\frac{n+3}{n+5} > 0$$

$$(n+3)\cdot(n+5) > 0$$

$n = -3 \quad n = -5$

I check if an algebraic function is greater than zero. To do that, I have to factorize it and draw a graph, that illustrates the situation. The graph should include the fact that n starts from 1. On the graph we can easily notice that $a_n > 0$



2° $\{a_n\} \downarrow$?

2° - the second condition is: Is the a_n a decreasing sequence?

In this step we have to check if $a_{n+1} - a_n$ is greater then zero or smaller than zero. [if the sequence is increasing or decreasing]

$$a_{n+1} - a_n = \frac{n+4}{n+6} - \frac{n+3}{n+5} =$$

$$= \frac{(n+4)(n+5) - (n+6)(n+3)}{(n+6)(n+5)} =$$

$$= \frac{n^2 + 5n + 4n + 20 - n^2 - 18 - 6n - 3n}{(n+6)(n+5)} = \frac{20-18}{(n+6)(n+5)} = \frac{2}{(n+6)(n+5)} > 0$$

$a_{n+1} - a_n > 0$ so $a_{n+1} > a_n$ $\rightarrow$ the sequence is increasing

$\{a_n\} \nearrow$

The condition is not fulfilled!

3° $\lim_{n \to \infty} a_n \overset{?}{=} 0$

3° In the third condition we need to check if the limit of a_n is zero.

$$\lim_{n\to\infty} \frac{n+3}{n+5} = \lim_{n\to\infty} \frac{\cancel{n}\left(1+\overbrace{\frac{3}{n}}^{\to 0}\right)}{\cancel{n}\left(1+\underbrace{\frac{5}{n}}_{0}\right)} = 1$$

In this step we can extract n in front of the bracket in numerator and denominator and we get a limit of $\frac{1}{1}$ which is 1

This condition wasn't fulfilled, because $\lim_{n\to\infty} a_n \neq 0$

After doing the alternating test we can say that the series $\sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{n+3}{n+5}$ is DIVERGENT, because the second and third condition weren't fulfilled.

We don't have to continue the test.

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