AZ gr. 3.

What to say in front of the blackboard - a brief tutorial.

Establish the convergence of $\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{2+n}{n}\right)^{n^2}$

$$\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{2+n}{n}\right)^{n^2}$$

the summation from 1 to infinity of terms: minus 1 to the nth plus 1 power times 2 plus n over n to the n-th squared power

$$a_n = \left(\frac{2+n}{n}\right)^{n^2}$$

I notice that it's alternating series, so I will consider a_n to be only 2 plus n over n to the n th squared power

We need to perform alternating series test, which consist of - conditions:

1° $a_n > 0$?

1° we check if a_n is greater than 0

Since n belongs from 1 to infinity and there are no negative factors in a_n, we know that if we substitute any number from 1 to infinity as n, we will always get a positive value

1° is fulfilled

2° $\{a_n\} \downarrow$?

$$a_n = \left(\frac{2+n}{n}\right)^{n^2}$$

$\{a_n\} \nearrow$

2° we have to check if a_n sequence is decreasing

We see that 2 plus n grows faster than n, so the sequence a_n is increasing

Because the second condition is not fulfilled, we know that

$\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{2+n}{n}\right)^{n^2}$ is divergent.