- What to say in front of the blackboard - a brief tutorial

Exercise: Using integral test, check whether the series converges or diverges of $\sum_{n=1}^{\infty} \frac{e^{\sqrt{n}}}{\sqrt{n}}$

Summation, from 1 to infinity of terms e to the power of root of n over root of n

In following steps we have to check if series finite or not.
Series converges if and only if the integral is finite. In particular, if the integral diverges, then series diverges as well.

$$\sum_{n=1}^{\infty} \frac{e^{\sqrt{n}}}{\sqrt{n}} = \lim_{a\to\infty} \int_1^a \frac{e^{\sqrt{n}}}{\sqrt{n}}\, dn =$$

In first step we introduce sign of lim because we are looking for limit of this series. We also change interval notation, in stead of ∞ sign we introduce letter a.

$$= \left\{ \begin{array}{l} \sqrt{n} = t \\ n = t^2 \\ dn = 2t\,dt \\ 1 \to \sqrt{1} \to 1 \\ \infty \to \sqrt{\infty} \to \infty \end{array} \right\} =$$

To solve our example I decided to use substitution. The $\sqrt{n}$ I substitute to t. Implication of previous substitution is changing n to t^2. Dn is a substitute to 2tdt. In last rows I show procedure during changing intervals, but in our case they stay the same.

$$= \lim_{a\to\infty} \int_1^{\sqrt{a}} \frac{e^t}{t} \cdot 2t\,dt =$$

$$= \lim_{a\to\infty} 2 \int_1^{\sqrt{a}} \frac{e^t}{\cancel{t}} \cancel{t}\,dt =$$

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We can put 2 before the integral and cross off letter t.

$$= 2 \cdot e^t \Big|_1^{\infty} = 2 \cdot [e^{\infty} - e^1] =$$

$$= 2 \cdot [\infty - e] = \infty$$

In next step we are integrating our function, but in our case remain the same. Then we will substitute t to a (∞) and to 1 (due to interval of the integral.

When we take $e^{\infty} = \infty$ and minus $e^1 = e$ we obtain ∞.

Result obtained from our exercise is ∞ which means it is infinite.

We can say that series $\sum_1^{\infty} \frac{e^{\sqrt{n}}}{\sqrt{n}}$ is DIVERGENT

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