1R WHAT TO SAY IN FRONT OF THE BLACKBOARD ?

$$\sum_1^\infty \frac{(-1)^n \cdot n}{2^n}$$

Summation, from 1 to infinity of terms: minus one to the nth power times n over two to the n-th power.

I/ $a_n = \frac{n}{2^n}$

I notice that it's an alternating series, so I will consider a_n to be only n over two to the nth power (without the $(-1)^n$ part)

We need to perform the alternating series test. It involves checking three conditions.

1° $a_n > 0$?

$\frac{n}{2^n} > 0$

$n 2^{-n} > 0$

1° is a_n greater than zero for all $n \geq 1$?

I check if an algebraic function is greater than zero and it is.

2° $\{a_n\} \searrow$?

$a_{n+1} < a_n$

2° is a_n a decreasing sequence? It is easy to see that a_n decreases because n will grow slower than the denominator 2 to the power of n.

3° $\lim_{n\to\infty} a_n = 0$?

$\lim_{n\to\infty} \frac{n}{2^n} = 0$?

$\lim_{x\to\infty} \frac{x}{2^x} \overset{H}{=} \lim_{x\to\infty} \frac{1}{\underbrace{2^x \ln 2}_{\infty}} = \frac{1}{\infty} = 0$

3° In the third condition we need to check if the limit of a_n is 0. We use the de l'hospital's rule using derivatives and we get limit equal to zero.

So, the series $\sum_1^\infty \frac{(-1)^n n}{2^n}$ is convergent.

II/ Now, let's take a look at a similar series:

$\sum_1^\infty \left| \frac{(-1)^n \cdot n}{2^n} \right|$.

$a_n = \left| \frac{(-1)^n \cdot n}{2^n} \right| = \frac{n}{2^n}$

Here, a_n is the same as in the step I, but this is no longer alternating series.

We will use the ratio test.

$q = \lim_{n \to \infty} \frac{a_{n+1}}{a_n}$

$a_n = \frac{n}{2^n} \qquad a_{n+1} = \frac{n+1}{2^{(n+1)}} = \frac{n+1}{2^n \cdot 2}$

$q = \lim_{n \to \infty} \frac{n+1}{\cancel{2^n} \cdot 2} \cdot \frac{\cancel{2^n}}{n} = \lim_{n \to \infty} \frac{n+1}{2n} =$

$\lim_{n \to \infty} \frac{\cancel{n}(1 + \overbrace{\tfrac{1}{n}}^{=0})}{2\cancel{n}} = \frac{1}{2}$

$q = \frac{1}{2} \qquad \frac{1}{2} < 1$

We get the value of q equal to $\frac{1}{2}$, so the series is convergent.

Since $\sum_{1}^{\infty} \frac{(-1)^n \cdot n}{2^n}$ is CONVERGENT

and $\sum_{1}^{\infty} \left| \frac{(-1)^n \cdot n}{2^n} \right|$ is CONVERENT

then $\sum_{1}^{\infty} \frac{(-1)^n \cdot n}{2^n}$ is ABSOLUTELY CONVERGENT

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