What to say in front of the blackboard – a brief tutorial

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Exercise: Establish the convergence (none, absolute, conditional) of: $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n^2}}$

Summation from 1 to infinity of terms: minus 1 to the n^{th} power over cube root of n to 2th power.

$1/$ $a_n = \frac{1}{\sqrt[3]{n^2}}$

I notice that it's an alternating series, so I will consider a_n to be only 1 over cube root of n to the 2 nd power (without the $(-1)^n$ part)

The check, if our example is convergent we must perform the alternating series test. It involves 3 conditions.

$1°$ $a_n > 0$

$$\frac{1}{\sqrt[3]{n^2}} > 0$$

$n = 1, 2, 3$

1° is a_n greater than zero for all $n \geq 1$?

The a_n is greater than zero we can show it when we substitute all numbers which are ≥ 1 then in all cases a_n will be positive.

$2°$ $\{a_n\} \searrow$

2° The second condition is: a_n is a decreasing sequence. a_n decreases because we're dividing 1 by numbers that get larger and larger.

$3°$ $\lim_{n\to\infty} \frac{1}{\sqrt[3]{n^2}} = 0$

$\Rightarrow \infty$

3° In the third condition we need to check if the limit of a_n is zero.

Our n to the power of 2 tends to the infinity, so when we devide 1 over infinity we obtain 0. So it's CONVERGENT.

convergent

$$|a_n| = \left|\frac{(-1)^n}{\sqrt[3]{n^2}}\right| = \frac{1}{\sqrt[3]{n^2}} = \frac{1}{n^{\frac{2}{3}}}$$

Now we check convergence of $|a_n|$ appropiate method is the p-series.

$p = \frac{2}{3}$

$p \leq 1$

divergent

If $p \leq 1$ so in our case series is divergent.

$$\sum_{1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n^2}}$$

$\sum_{1}^{\infty} a_n$ is convergent but $\sum_{1}^{\infty} |a_n|$ is not convergent, so $\sum_{1}^{\infty} a_n$ is CONDITIONALLY CONVERGENT

Since $\sum_{1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n^2}}$ is convergent

and $\sum_{1}^{\infty} \left| \frac{(-1)^n}{\sqrt[3]{n^2}} \right|$ is divergent,

the $\sum_{1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n^2}}$ is conditionally convergent