Having a given series: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\cdot n}{2n^2+3}$ we need to determine our a_n what is equal to: $\frac{n}{2n^2+3}$.

Next step is deciding that a_n is positive or negative. In this case a_n is greater than zero ($a_n > 0$).

Furthermore, we have to establish whether our a_n is increasing or decreasing. We see that a_n is decreasing because $2n^2+3$ grows faster than n.

Then, we should check convergence/divergence by calculating the limit of a_n.

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\frac{n}{2n^2+3} = \lim_{n\to\infty}\frac{\cancel{n}}{\cancel{n}(2n+\underbrace{\tfrac{3}{n}}_{0})} = \frac{1}{2n} = \frac{1}{\infty} = 0$$

First of all, in our limit we need to separate n before the parenthesis. in the denominator and cross out our n. It gives us a simplified expression. Whe know that three devided by n is zero, so we have only one over two times n and again we must devide a number one by infinity (two times n). Our result is zero what means a_n is CONVERGENT.

The essence of the last step is determining whether a_n is none, absolutely or conditionally convergent.

We use the integral:

$$\int_1^{\infty}\frac{x}{2x^2+3}\,dx = \lim_{A\to\infty}\int_1^{A}\frac{x\,dx}{2x^2+3} = \left\{\begin{array}{l} t = 2x^2+3 \\ dt = 4x\,dx \\ \frac{dt}{4} = x\,dx \\ 1 \to 2+3=5 \\ A \to 2A^2+3 \end{array}\right\} = \lim_{A\to\infty}\frac{1}{4}\int_5^{2A^2+3}\frac{dx}{x} = \lim_{A\to\infty}\frac{1}{4}\cdot \ln x\Big|_5^{2A^2+3} =$$

$$= \lim_{A\to\infty}\frac{1}{4}\left(\ln(\underbrace{2A^2+3}_{\infty}) - \ln 5\right) = \infty \rightarrow \text{DIVERGENT}$$

In our integral we assume A is equal to infinity. Next we substitute $2x^2+3$ as t and calculate derivative of it. Furthermore, we ~~are~~ compute t supposing x is equal to A and 1. Then, we must take out $\frac{1}{4}$ before our integral (we see our limits are changed: $2A^2+3$ and 5). The last part is counting our fraction by substituting x by the limits. The result is infinity what confirms the divergence.

To sum up, that series is only CONVERGENT CONDITIONALLY!

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