

What to say in front of the blackboard - a brief lecture

Exercise: Establish the convergence (none, absolute, conditional) of:

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[3]{n^4+1}}$$

Summation, from 1 to infinity of terms: minus one, to the $n+1$ power over cube root of n to fourth power plus one.

$$a_n = \frac{1}{\sqrt[3]{n^4+1}}$$

I notice that it's an alternating series, so I'll consider a_n to be only one over cube root of n to fourth power plus one.

To check our example is convergent. We must perform the alternating series test. It involves checking three conditions.

1°

$$a_n > 0 \quad ?$$

1° is a_n greater than zero for all $n \geq 1$?

$$\frac{1}{\sqrt[3]{n^4+1}} > 0$$

$$n = 1, 2, 3, \dots$$

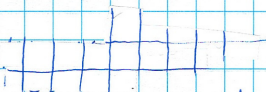
In our case, we can say yes, because when we put for n some numbers ≥ 1 than every solution will be positive.

2°

$$|a_n| \searrow$$

2° The second condition is: Is a_n a decreasing sequence. We can see fast, that a_n is decreasing, because we're dividing one by larger numbers.

3°


$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[3]{n^4+1}} = 0$$

3° In the third condition we need to check if limit of a_n equal zero. Our n to the fourth power tends to infinity, so when we divided one over infinity we get 0.

All three conditions are satisfied so is CONVERGENT

Now we check if our series is absolutely, conditionally (or none) convergent:

$$|a_n| = \left| \frac{(-1)^{n+1}}{\sqrt[3]{n^4+1}} \right| = \frac{1}{\sqrt[3]{n^4+1}}$$

$$\sqrt[3]{n^4+1} > \sqrt[3]{n^4}$$

$$\frac{1}{\sqrt[3]{n^4+1}} < \frac{1}{\sqrt[3]{n^4}}$$

$$\frac{1}{n^{\frac{4}{3}+1}} < \frac{1}{n^{\frac{4}{3}}}$$

Here a_n is the same as in step 1, but this is no longer an alternating series - we will use comparison test.

We simplified our sequence and now look to our power, if it's greater than 1, we search series which will be convergent, so greater equal to our series.

From the p-series theory we know that the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent. (here $p = \frac{4}{3} > 1$)

So our original series $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^4+1}}$ must be convergent as well.

Since $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[3]{n^4+1}}$ is convergent

$\sum_{n=1}^{\infty} \left| \frac{(-1)^{n+1}}{\sqrt[3]{n^4+1}} \right|$ is convergent

the $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[3]{n^4+1}}$ is **ABSOLUTELY CONVERGENT**

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