What to say in front of the blackboard - a brief tutorial

Exercise: Establish the convergence (none, absolute, conditional) of:

$$\sum_{1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[3]{n^4+1}}$$

Summation, from 1 to infinity of terms: minus one, to the $n+1$ power over cube root of n to fourth power plus one

I. $a_n = \frac{1}{\sqrt[3]{n^4+1}}$

I notice that it's an alternating series, so I'll consider a_n to ~~be~~ only one over cube root of n to fourth power plus one.

To check our example is convergent. We must perform the alternating series test. It involves checking three conditions.

1°

$a_n > 0$?

1° is a_n greater than zero for all $n \geq 1$?

$$\frac{1}{\sqrt[3]{n^4+1}} > 0$$

$n = 1, 2, 3 \ldots$

In our case, we can say yes, because, when we put for n some numbers ≥ 1 than every solution will be positive.

2°

$\{a_n\} \searrow$

2° The second condition is: Is a_n a decreasing sequence. We can see fast, that a_n is decreases, because we're dividing one by larger numbers.

3°

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{\sqrt[3]{\underbrace{n^4+1}_{\to\infty}}} = 0$$

3° In the third condition we need to check is limit of a_n equal zero. Our n to the fourth power tends to infinity, so when we divided one over infinity we get 0.

All three conditions are satisfied so is <u>CONVERGENT</u>

Verte

II Now we check is our series is absolutelly, conditionally (or none) convergent:

$$|a_n| = \left|\frac{(-1)^{n+1}}{\sqrt[3]{n^4+1}}\right| = \frac{1}{\sqrt[3]{n^4+1}}$$

Here a_n is the same as in step I, but this is no longer an alternating series - we will use comparison test.

$$\sqrt[3]{n^4+1} > \sqrt[3]{n^4}$$

We simplified our sequence and now look to our power, if is greater than 1, we search series which will be convergent, so greater equal to our series.

$$\frac{1}{\sqrt[3]{n^4+1}} < \frac{1}{\sqrt[3]{n^4}}$$

$$\frac{1}{n^{\frac{4}{3}}+1} < \frac{1}{n^{\frac{4}{3}}}$$

From the p-series theory we know, that the series $\sum_{1}^{\infty} \frac{1}{\sqrt[3]{n^4}}$ is convergent.
(here $p = \frac{4}{3} > 1$)

So our original series $\sum_{1}^{\infty} \frac{1}{\sqrt[3]{n^4+1}}$ must be convergent as well.

Since $\sum_{1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[3]{n^4+1}}$ is convergent

$\sum_{1}^{\infty} \left|\frac{(-1)^{n+1}}{\sqrt[3]{n^4+1}}\right|$ is convergent

the $\sum_{1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[3]{n^4+1}}$ is ABSOLUTELY CONVERGENT

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