A BRIEF TUTORIAL - what to say in front of the blackboard...

Exercise: Check if the series is absolutely or conditionaly convergent or divergent.

$$\sum_{n=1}^{\infty} (-1)^n \frac{n+1}{2n-1}$$

Summation, from 1 to infinity of terms: minus 1 to the n-th power times n plus 1 over two times n minus 1

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$$a_n = \frac{n+1}{2n-1}$$

I notice that it's an alternating series, so I will consider a_n to be only n plus 1 over two times n minus 1 (without the $(-1)^n$ part)

We need to perform the alternating series test. It involves checking three conditions:

1° $a_n > 0$?

1° is a_n greater than zero for all $n \geqslant 1$?

$$\frac{n+1}{2n-1} > 0$$

$$(n+1)(2n-1) > 0$$

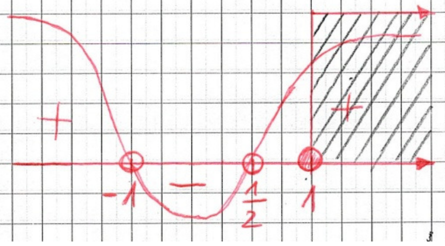
$$2(n+1)\left(n-\frac{1}{2}\right) > 0$$

$$\Downarrow \qquad \Downarrow$$

$$n = -1 \qquad n = \frac{1}{2}$$

It's an exercise from the first semester:

- I check if an algebric function is greater than 0.
I need to factorise 2 in front of the bracket. Then I draw the graph and include the fact than n starts from 1.



It's clear from the graph that:

$$a_n > 0$$

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2° $\{a_n\} \downarrow ?$

The second condition is:

2° is an a decreasing sequence?

It's easy to see that a_n decreases, because $(2n-1)$ is greater than $n+1$ and then the denominator increases faster than the numerator.

a_n is a decreasing sequence

3° $\lim\limits_{n\to\infty} a_n \overset{?}{=} 0$

In the third condition we need to check:

3° if the limit of a_n is zero?

This is also an exercise from the first semester.

$$\lim_{n\to\infty}\frac{n+1}{2n-1} = \lim_{n\to\infty}\frac{\cancel{n}\left(1+\overset{\overset{0}{\uparrow}}{\frac{1}{n}}\right)}{\cancel{n}\left(2-\underset{\underset{0}{\downarrow}}{\frac{1}{n}}\right)} = \frac{1}{2}$$

It's enough to extract n in front of the bracket in the numerator and denominator. We get a limit one over two.

I know that if $\lim\limits_{n\to\infty} a_n \neq 0$ then the series is: DIVERGENT.

The third condition wasn't fulfilled so I don't have to continue the test.

The series $\sum\limits_{n=1}^{\infty} (-1)^n \frac{n+1}{2n-1}$ is DIVERGENT

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