A BRIEF TUTORIAL

What to say in front of the BlackBoard.

EXERCISE:
Check if the series is absolutely or conditionaly convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n \cdot 3^n}$$

The summation from 1 to infinity of terms minus 1 to the n-th power over n times 3 to the n-th power.

$$a_n = \frac{1}{n \cdot 3^n}$$

I notice that it's alternating series, so I consider a_n as 1 over n times 3 to the n-th power.

We need to perform alternating series test, it consists of 3 conditions.

$$1^\circ \quad a_n > 0 \ ?$$

1° We check if a_n is greater than 0.

$$n \in \langle 1, +\infty)$$

$$\frac{1}{n \cdot 3^n} > 0$$

Since there are no negative factors in our expression: one over n times 3 to the n-th power, and n belongs from 1 to infinity, we see at the first glance that all a_n will be greater then 0.
1° condition is fullfilled.

$$2^\circ \quad \{a_n\} \downarrow ?$$

2° We check if the sequence of a_n is decreasing.

$$\frac{1}{n \cdot 3^n} \longrightarrow 0$$

$$\Downarrow$$

$$\{a_n\} \downarrow$$

We see that the numerator, equals 1, is constant, while the denominator will grow.
According to that we know that term one over n times 3 to the n-th power, will tend to 0.
The sequence of a_n is decreasing.
2° condition is fullfilled.

$$3^\circ \quad \lim_{n \to \infty} a_n \overset{?}{=} 0$$

3° We check if the limit of a_n is equal 0.

$$\lim_{n \to \infty} \frac{1}{\underbrace{n \cdot 3^n}_{\to \infty}} \overset{\left[\frac{1}{\infty}\right]}{=} 0$$

The n times 3 to the n-th power tends to infinity, when n tends to infinity. If we devide 1 by infinity many times, it will tend to 0, so the limit equals 0.
3° condition is fullfilled.

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{n \cdot 3^n}$$

is <u>convergent</u>

All conditions for an alternating series are fullfilled, so the series of minus 1 to the n-th power times 1 over n times 3 to the n-th power is convergent.

Now let's check if the series is convergent absolutely or conditionaly.
To check that I will perform ratio test on absolute value of a_n.

$$|a_n| = \left|(-1)^n \frac{1}{n \cdot 3^n}\right| = \frac{1}{n \cdot 3^n}$$

The absolute value of a_n is 1 over n times 3 to the n-th power.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n}$$

In ratio test we determine the limit of a_{n+1} over a_n.

$$a_n = \frac{1}{n \cdot 3^n}$$

$$a_{n+1} = \frac{1}{(n+1)\cdot 3^{n+1}} = \frac{1}{3n \cdot 3^n + 3 \cdot 3^n} = \frac{1}{3^n(3n+3)}$$

I substitute n+1 in place of n and I simplify the term a_{n+1}.

Our final a_{n+1} is 1 over 3 to the n-th power times 3n plus 3.

$$\lim_{n\to\infty} \frac{1}{\cancel{3^n}(3n+3)} \cdot \frac{n \cdot \cancel{3^n}}{1} = \lim_{n\to\infty} \frac{\cancel{n}\,1}{\cancel{n}(3 + \underbrace{\tfrac{3}{n}}_{0})} = \frac{1}{3}$$

$$\frac{1}{3} < 1$$

We have the limit of 1 over 3 to the n-th power times 3n plus 1 times the reverse of a_n, which is n times 3 to the n-th power over 1.

We simplify 3 to the n-th power and we factor out n in the denominator.
We get the limit of 1 over 3 plus 3 over n. 3 over n tents to 0.
The limit is equal one third.

The series

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{n \cdot 3^n}$$

is

<u>absolutely convergent.</u>

Since one third is less than 1, we know that series from 1 to infinity of terms minus 1 to the n-th power times 1 over n times 3 to the n-th power is absolutely convergent.

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