Experiment 1

DETERMINATION OF YOUNG'S MODULUS BY THE RESONANCE METHOD

The aims of the experiment are to study the frequency dependence of the forced oscillations of a short solid bar and to determine Young's modulus of a solid by measuring the resonance frequency.

INTRODUCTION

1. The speed of propagation of longitudinal as well as transverse waves is determined by the mechanical properties of the medium.

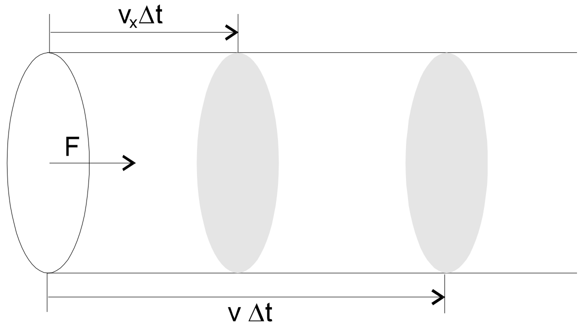

Fig. 1.1. Illustration of the compression of a solid bar by force F

Figure 1.1 shows an end–section of a solid bar with density ρ and cross–sectional area A. At t=0 a force F is applied to one end of the bar. It makes the bar compress at the end and the end portion of the bar moves with speed v_x. It also initiates a wave motion that travels with a speed v to the right along the bar. After time Δt the wave has moved a distance $v\Delta t$, but the left end of the bar has compressed by $v_x\,\Delta t$.

The speed of propagation of the wave can be computed from the impulse–momentum theorem

$$F\,\Delta t = \Delta p \tag{1.1}$$

The change of momentum Δp is calculated for the cylinder with length $v\Delta t$ and cross–sectional area A set in motion at time Δt with a speed v_x

$$\Delta p = A\, v\, \Delta t\, \rho\, v_x \tag{1.2}$$

It has been assumed that all parts of the cylinder have the same speed v_x. The force F, producing the contraction of the bar is computed from Hooke's law. The original length of the cylinder $v\,\Delta t$ has decreased by an amount $v_x\,\Delta t$ and the pressure $\frac{F}{A}$ exerted on the bar is

$$\frac{F}{A} = E\,\frac{v_x \Delta t}{v \Delta t} \tag{1.3}$$

where E is Young's modulus. The impulse is

$$F\,\Delta t = E\,\frac{v_x}{v}\,A\,\Delta t \tag{1.4}$$

Applying the impulse–momentum theorem it is found

$$A\, v\, \Delta t\, \rho\, v_x = E\,\frac{v_x}{v}\,A\,\Delta t \tag{1.5}$$

and the speed of propagation v of the longitudinal wave is given by

$$v = \sqrt{\frac{E}{\rho}} \tag{1.6}$$

The speed depends only on the Young's modulus and the density of the medium.

2. When a longitudinal wave propagates in a solid bar with finite length, the wave is reflected from the ends. The superposition of incoming and reflected waves traveling in opposite directions forms a standing wave. If the bar is rigidly held in the middle by a support both free ends must be antinodes and the middle must be a node with adjacent nodes being one half wavelength $\frac{\lambda}{2}$ apart (figure 1.2).

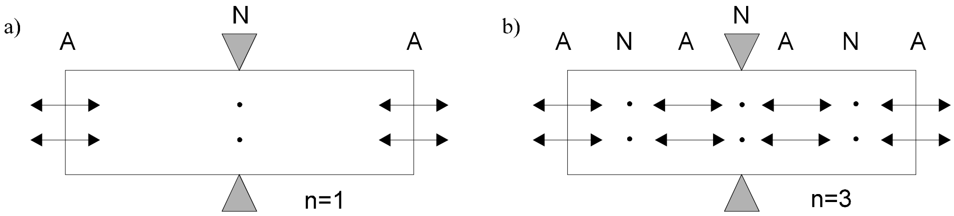

Fig. 1.2. Standing waves in a bar held rigidly in the middle by a support.
Nindicates nodes and A antinodes

The wavelength, λ, must therefore be related to the length of the bar, L, by

$$L = n\frac{\lambda}{2} \qquad (n = 1, 3, 5,...) \tag{1.7}$$

where n is an odd integer number. Solving this equation for λ, the following possible values of the wavelength λ_n are obtained

$$\lambda_n = \frac{2L}{n} \qquad (n = 1, 3, 5,...) \tag{1.8}$$

Corresponding to these wavelengths are the possible frequencies f_n of the oscillations in the bar

$$f_n = \frac{v}{\lambda_n} = \frac{v}{2L}n \qquad (n = 1, 3, 5,...) \tag{1.9}$$

where v is the velocity of propagation of the wave. The smallest frequency f_1 corresponding to n = 1 (figure 1.2a) where,

$$f_1 = \frac{v}{2L} \tag{1.10}$$

is called the fundamental frequency. The other frequencies f_3, f_5,... are integer multiples of f_1 and are called harmonics.

3. Mechanical systems (for example a solid bar) have normal modes of oscillations. In each mode all particles of the system oscillate with simple harmonic motion with the same frequency. In the previous section it has been shown that the standing wave oscillations of a bar have an infinite series of normal mode frequencies f_n. If a periodically varying force is applied to a system it is forced to vibrate with a frequency equal to the frequency of the

force. This vibrational motion of the system is greatest when the frequency of the force is equal to one of the normal mode frequencies. The system then is in mechanical resonance with the external force. For other values of the force frequency the amplitude of the motion is in general relatively small. Figure 1.3 shows a graph of the amplitude a of forced oscillation as a function of frequency f of the force. This curve is called a resonance curve. It reaches a maximum at a normal mode frequency f_n.

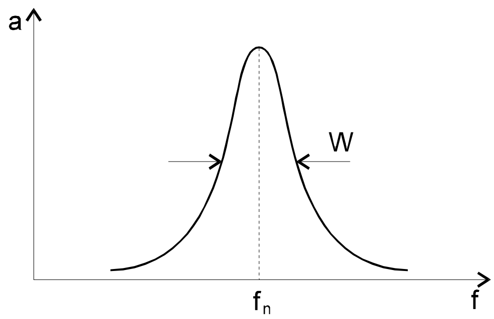

Fig. 1.3. Resonance curve: a graph of the amplitude a as a function of frequency f. W is the width of the resonance curve

The width of the resonance curve W is measured at an amplitude equal to half of its maximum value. The width W is small giving a sharply peaked resonance curve when there is little friction or other energy dissipation in the system.

APPARATUS AND METHOD

Young's modulus is obtained from the expression

$$E = v^2 \rho \tag{1.11}$$

which is derived from (1.6). The velocity v is obtained from measurements of the fundamental frequency f_1 by a resonance method. It is next calculated from

$$v = 2\, f_1\, L \tag{1.12}$$

where L is the length of the solid bar.

The experimental setup used to measure the fundamental frequency is shown schematically in figure 1.4 It consist of a solid bar supported horizontally in the middle of its length by a clamp, a driving coil connected to an audio signal generator and a pick-up coil connected to an oscilloscope. A small ferromagnetic disc is attached to each end of the solid bar. This allows the production of oscillations in the bar by the driving coil and also the detection of the movement of the other end of the bar by the pick-up coil. The amplitude of the oscillations is measured by observing the signal on the screen of the oscilloscope. In the experiment the amplitude of the signal on the screen is measured as a function of the frequency of the signal generator to obtain the resonance curve and to determine the frequency of the fundamental mode f_1..

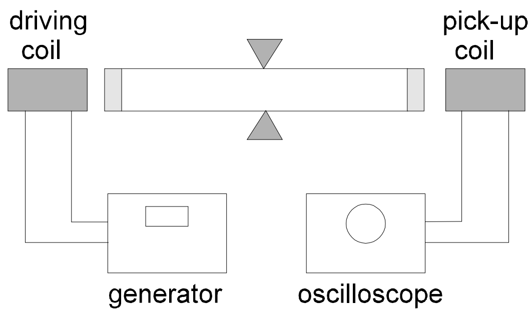

Fig. 1.4. Schematic diagram of the experimental setup used in the measurements of the fundamental frequency of the oscillations of the solid bar

MEASUREMENTS

1. Measure the amplitude of oscillations as a function of frequency for the three bars provided which are made of steel, copper and brass. Change the frequency in steps of 1Hz in the range from 3405Hz to 3435Hz for the brass bar, from 3855Hz to 3885Hz for the copper bar and from 5190Hz to 5220Hz for the steel bar. Plot a graph for each set of data to obtain the resonance curve. From the curve find the frequency of the fundamental mode f_1 and the resonance curve width W for each bar.
2. Calculate the velocity of sound and Young's modulus for the three materials measured (steel, copper and brass). The densities of the above materials are as follows:

   steel - $(7.88 \pm 0.01) \times 10^3 kg/m^3$
   copper - $(8.81 \pm 0.01) \times 10^3 kg/m^3$
   and brass - $(8.70 \pm 0.01) \times 10^3 kg/m^3$.

   Compare the obtained values with that given in table 4 and table 5.

ANALYSIS OF ERRORS

The errors in the measurements of the velocity, Δv, and Young's modulus, ΔE, are calculated from the expressions

$$\Delta v = 2\ L\ \Delta f + 2\ f\ \Delta L \tag{1.13}$$

and

$$\Delta E = 2\ v\ \rho\ \Delta v + v^2 \Delta\rho \tag{1.14}$$

obtained from the differentiation of (1.11) and (1.12). Here Δf is the error in measurement of the frequency, ΔL of the bar length and $\Delta\rho$ of the density of bar material.

QUESTIONS

1. Give the detail of the derivation of equation (1.6) for the velocity of the wave.
2. Explain the appearance and properties of a standing wave.
3. What are the harmonic frequencies of vibration of a string held at its two ends.
4. Energy can be transferred by waves. Explain why standing waves can not transfer energy.