Experiment 3

DETERMINATION OF THE MOMENT OF INERTIA OF A SOLID

In this experiment the moments of inertia of various rigid bodies are determined and are compared with the calculated values.

INTRODUCTION

When a rigid body rotates about a fixed axis the speed of a particle with mass, m_i, which is at distance, r_i, from the axis of rotation is given by, $v_i = r_i\,\omega$. ω is the angular velocity of the body and is the same for all particles. The kinetic energy of the particle, E_k^i, is given by,

$$E_k^i = \frac{1}{2} m_i v_i^2 = \frac{1}{2} m_i r_i^2 \omega^2 \tag{3.1}$$

The kinetic energy of the body is the sum of the kinetic energies of all particles

$$E_k = \sum_i E_k^i = \frac{1}{2}\left(\sum_i m_i r_i^2\right)\omega^2 \tag{3.2}$$

The quantity in parentheses is called the moment of inertia of the body I, where

$$I = \sum_i m_i r_i^2 \tag{3.3}$$

and it is the sum of the products for all particles of the mass of each particle by the square of its distance from the axis of rotation.The rotational kinetic energy of a rigid body is then

$$E_k = \frac{1}{2} I\,\omega^2 \tag{3.4}$$

This is an analogous expression to $E = \frac{1}{2} mv^2$ for the kinetic energy of a body in linear motion. The moment of inertia is analogous to mass in a similar way as angular velocity is analogous to velocity v.

When the body is a continuous distribution of matter, as for example in a solid sphere, the sum in (3.3) is replaced by an integral. Then the moment of inertia is

$$I = \int r^2 dm \tag{3.5}$$

where dm is a small mass element of the body. The limits on the integral are determined by the dimensions of the body. The integration can be carried out easily for bodies of regular shape by representing r and dm in terms of integration variables. In figure 3.1 a solid rod and a solid sphere rotating about an axis through the center of mass are shown and their moments of inertia are given in terms of their masses M and dimensions.

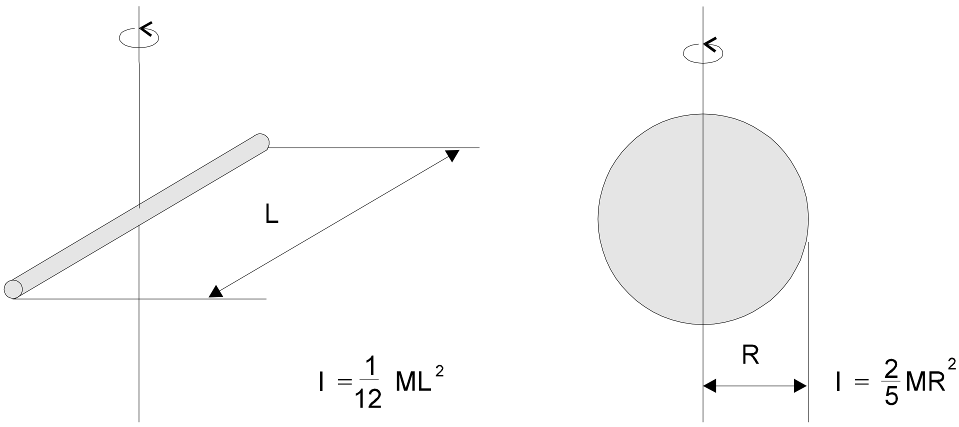

Fig. 3.1. Moments of inertia of a solid rod of legth L and mass M and of a sphere of radius R and mass M

The rotational motion of a rigid body is described by a fundamental dynamical equation which is analogous to the Newton's second law of motion

$$\tau = I\,\alpha \tag{3.6}$$

The angular acceleration, $\vec{\alpha} = \frac{d\omega}{dt}$, of a rotating body is directly proportional to the total torque τ acting on the body. The moment of inertia, I, is the proportionality factor.

APPARATUS AND METHOD

The moment of inertia of a rigid body is measured using an apparatus shown in figure 3.2. A weight of mass m is attached to a fine thread which is wrapped around bobbin S.

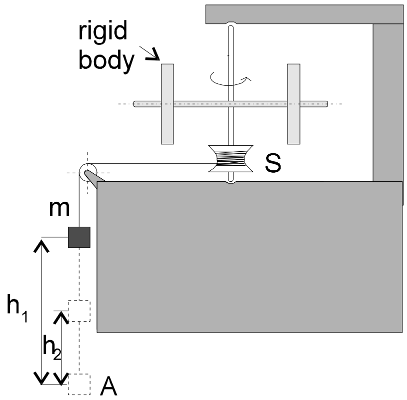

Fig. 3.2. Schematic diagram of an apparatus used to measure the moment of inertia of a rigid body

The weight is allowed to fall through a distance, h_1 , to its lowest position A. The time of the descent t is measured by a stop-watch and also the the number of revolutions n of the body during time t is taken. At point A the kinetic energy of the rotating body is greatest and now it rises the weight up through a final distance h_2. The distance h_2 is lower then h_1 due to the friction in the apparatus. The experiment is repeated 3 times for the same distance h_1 and average values of t, n and h_2 are calculated.

From the principle of the conservation of energy

$$m\,g\,h_1 = \frac{1}{2} I\omega^2 + T\,h_1 \tag{3.7}$$

where $\frac{1}{2} I\omega^2$ is the rotational kinetic energy of the rigid body and T h_1 is the work done against friction T.

Similary for the rise of the weight

$$\frac{1}{2} I\omega^2 = m\,g\,h^2 + T\,h_2 \tag{3.8}$$

In both equations the kinetic energy of the weight has been neglected in comparison to the much higher rotational energy of the body. Substituting for T in (3.7) from (3.8) it is found that

$$I = 4mg\frac{h_1 h_2}{h_1 + h_2}\frac{1}{\omega^2} \tag{3.9}$$

The acceleration in the rotational motion is constant and the angular velocity, ω, is equal to twice the average velocity

$$\omega = 2\frac{2\pi n}{t} = \frac{4\pi n}{t} \tag{3.10}$$

Finally the moment of inertia is given by,

$$I = \frac{1}{4\pi^2}\frac{mgt^2}{n^2}\frac{h_1 h_2}{h_1 + h_2} \tag{3.11}$$

MEASUREMENTS

1. Find the moment of inertia of a horizontal solid bar.
2. Find the moment of inertia I_M of two weights of equal masses M placed at equal distances R from the axis of rotation from the measurements of the moment I_C for the solid bar with the attached weights. Use the relation

$$I_M = I_C - I_0 \tag{3.12}$$

   where I_0 is the moment of inertia of a solid bar.
3. Calculate the moments of inertia of the two weights and the solid bar and compare the obtained values with the results of the measurements. The mass of the bar is equal to 2kg.

ANALYSIS OF ERRORS

The error ΔI in the measurements of the moment of inertia I is calculated from the expression

$$\frac{\Delta I}{I} = \frac{\Delta m}{m} + \left(\frac{h_1}{h_2} + \frac{h_2}{h_1} \right) \frac{\Delta h}{h_1 + h_2} + 2\frac{\Delta t}{t} + 2\frac{\Delta n}{n} \tag{3.13}$$

where Δm is the error in the mass m of the weight, Δh is the error in the determination of distances h_1 and h_2, Δt is the error in measurement of the time t and Δn is the error in the measurement of the number of revolutions. Δh, Δt and Δn have to be determined during the measurements and calculations of the mean values of h, t and n.

The error ΔI_M of I_M is

$$\Delta I_M = \Delta I_C + \Delta I_0 \tag{3.14}$$

QUESTIONS

1. Give the kinematic equations for rotational motion with a constant angular acceleration.
2. When a wheel rotates with constant angular velocity do the points on the wheel have acceleration?
3. Recent satelite measurements show that the Earth's moment of inertia is 0.33 MR^2 which is less than expected from the $\frac{2}{5}MR^2$ formula for a sphere. What is the explanation of this fact?
4. Prove that for a thin plane sheet the moment of inertia I_0 about an axis through 0 (origin of the xy-plane) perpendicular to the plane is

$$I_0 = I_x + I_y \tag{3.15}$$

where I_x and I_y are the moments of inertia about the x- and y-axes respectvely (perpendicular-axis theorem).