Experiment 4

DETERMINATION OF THE COEFFICIENT OF RIGIDITY OF A WIRE

In this experiment the elastic shear deformation of thin wire is studied by a dynamical method due to Gauss and the coefficient of rigidity of the wire is measured.

INTRODUCTION

A shear deformation of a block of material (for example metal, ceramics) is shown in figure 4.1. The force F_t, tangential to a material surface S produces a deformation in which the surface a b c d moves to the position a'b'c'd'.

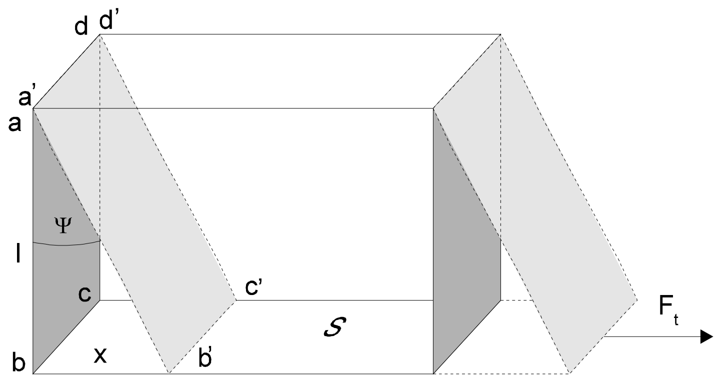

Fig. 4.1. Illustration of a shear deformation of a block of material due to atangential force F_t

The shear strain, σ, is a measure of the deformation and is defined as the ratio of the displacement x to the transverse length l

$$\sigma = \frac{x}{l} = \Psi \tag{4.1}$$

It is equal to the angle ψ (figure 4.1) because the displacement x is always much smaller than l.

For small forces which ensure that Hooke's law is obeyed the shear strain is proportional to the shear stress, p_t

$$p_t = \tau \Psi \tag{4.2}$$

The shear stress p_t is defined as the ratio

$$p_t = \frac{F_t}{S} \tag{4.3}$$

and τ is the coefficient of rigidity. For a given material τ is usually smaller than the corresponding Young's modulus.

When a torque is applied to the upper surface of a long cylindrical wire with a fixed position of the bottom surface it produces deformation of the wire which is shown in figure 4.2.

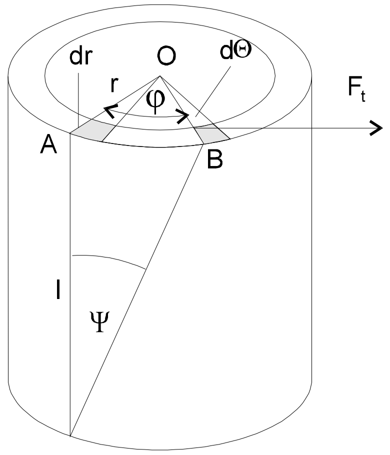

Fig. 4.2. Deformation of a long cylindrical wire due to the torque of a tangential force F_t

The shaded area, dS, bounded by circles of radii r and r+dr and radial lines from O under an angle $d\theta$ moves from position A to B when the upper surface is twisted through an angle φ. The whole cylinder is subject to a shear deformation with a shear strain

$$\Psi = \frac{r\varphi}{l} \tag{4.4}$$

The shear stress, p_t, for the area dS is given by

$$p_t = \tau \frac{r\varphi}{l} \tag{4.5}$$

and the torque, dM, about O by

$$dM = p_t dS\, r = \frac{\tau r^3 \varphi}{l} dr\, d\Theta \tag{4.6}$$

where dS = r dθ dr.
The total torque for the surface of a wire of a diameter d is obtained from the integration

$$M = \int_S dM = \frac{\tau\varphi}{l} \int_0^{\frac{d}{2}} \int_0^{2\Pi} r^3 dr\, d\Theta = \frac{\pi}{32} \frac{\tau d^4}{l} \varphi \tag{4.7}$$

It is seen from (4.7) that the torsion angle φ is proportional to the torque

$$M = D\varphi \tag{4.8}$$

where

$$D = \frac{\pi}{32} \frac{\tau d^4}{l} \tag{4.9}$$

APPARATUS AND METHOD

To measure the coefficient of rigidity of a wire Gauss's method is used where the period of oscillation of a solid symmetrical body attached to the wire (figure 4.3) is measured. If the body with moment of inertia I is displaced through a small angle φ it starts to oscillate around its equilibrium position with the motion described by

$$M = I\ddot{\varphi} \tag{4.10}$$

The torque M acting on the body is due to the restoring forces in the deformed wire and according to (4.8)

$$M = -D\varphi \tag{4.11}$$

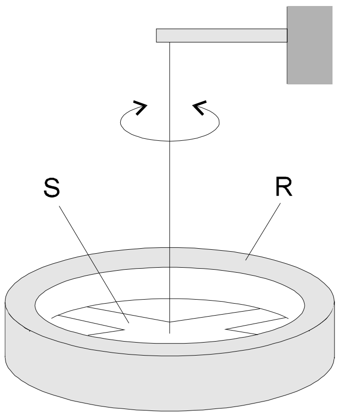

Fig. 4.3. Apparatus to measure the coefficient of rigidity of a wire. The wire is attached to a support S which holds a ring R

From (4.10) the equation of motion is

$$\ddot{\varphi} + \frac{D}{I}\varphi = 0 \tag{4.12}$$

Thus the motion is simple harmonic and its period is

$$T = 2\pi\sqrt{\frac{I}{D}} \tag{4.13}$$

In the experiment the period T_1 of the oscillations of the support S having moment of inertia I

$$T_1 = 2\pi\sqrt{\frac{I}{D}} \tag{4.14}$$

is measured. Then the ring R is placed on the support and the period T_2 is determined where

$$T_2 = 2\pi\sqrt{\frac{I + I_0}{D}} \tag{4.15}$$

Here I_0 is the moment of inertia of the ring. Substituting for I in (4.15) from (4.14) and using (4.9) the coefficient of rigidity can be obtained

$$\tau = 128\pi \frac{l}{d^4} \frac{I_0}{T_2^2 - T_1^2} \tag{4.16}$$

I_0 is calculated from

$$I_0 = \frac{1}{8} M\left(d_1^2 + d_2^2\right) \tag{4.17}$$

where M is the mass of the ring and d_1,d_2 are the internal and external diameters of the ring respectively.

MEASUREMENTS

1. Find the periods of oscillations T_1 and T_2 measuring the time of 20 complete oscillations. Repeat the measurements 10 times for each period. Calculate the mean values of the periods and their standard deviations (see (6) for the expression used to calculate the standard deviation).
2. Find the length l of the wire and its diameter d. Repeat each measurement 10 times and calculate mean values with their standard deviations.
3. Find the diameters d_1 and d_2 of the ring and calculate I_o from (4.17). The mass of the ring is equal to 1.1 kg.
4. Calculate the coefficient of rigidity of the wire using equation (4.16).

ANALYSIS OF ERRORS

The uncertainty in the measurement of τ is given as standard deviation σ_τ and is calculated using the expression

$$\frac{\sigma_\tau}{\tau} = \sqrt{\left(\frac{\sigma_l}{l}\right)^2 + \left(4\frac{\sigma_d}{d}\right)^2 + \left(\frac{2T_2}{T_2^2 - T_1^2}\sigma_{T_2}\right)^2 + \left(\frac{2T_1}{T_2^2 - T_1^2}\sigma_{T_1}\right)^2} \tag{4.18}$$

Here σ_l, σ_d, σ_{T1}, σ_{T2} are the standard deviations of l, d, T_1, and T_2, respectively. Formula (4.18) is obtained from (4.16) according to the rule of propagation of errors. The $(\sigma_{I_0} / I_0)^2$ term for the moment of inertia I_0 in (4.18) is neglected as it is much smaller than the remaining contributions.

QUESTIONS

1. Formulate Hooke's law and find what is the analogy with equation (4.2).
2. Draw a typical stress-strain diagram for a solid and explain the elastic and plastic deformation.
3. Is the coefficient of rigidity for rubber higher or lower then the same coefficient for steel.
4. Hooke's law applies to solids: what is the analogous law for liquids?