Experiment 5

MEASUREMENT OF THE VELOCITY OF SOUND INAIR

The velocity of sound in air is measured for different values of the frequency of the source using a resonance method. From the measurements the ratio of the molar heat capacity at constant pressure to that at constant volume, κ, for air is determined.

INTRODUCTION

1. Longitudinal waves propagating in a column of a gas in a pipe having a finite length are reflected from the end of the pipe and the superposition of the waves traveling in opposite directions produces a standing wave. When the reflection of the wave takes place at a closed end of the pipe, there is no displacement of the particles at the end and a node of the standing wave is formed. If the end of a pipe is open there is a free movement of particles at the end and an open end is an antinode.

The longitudinal standing waves in an air column of length L form a series of normal modes of vibration of air which differ by their wavelengths. If both ends of the pipe are open the normal modes of vibrations will have antinodes at the ends. This is illustrated in figure 5.1a for the first three normal modes.

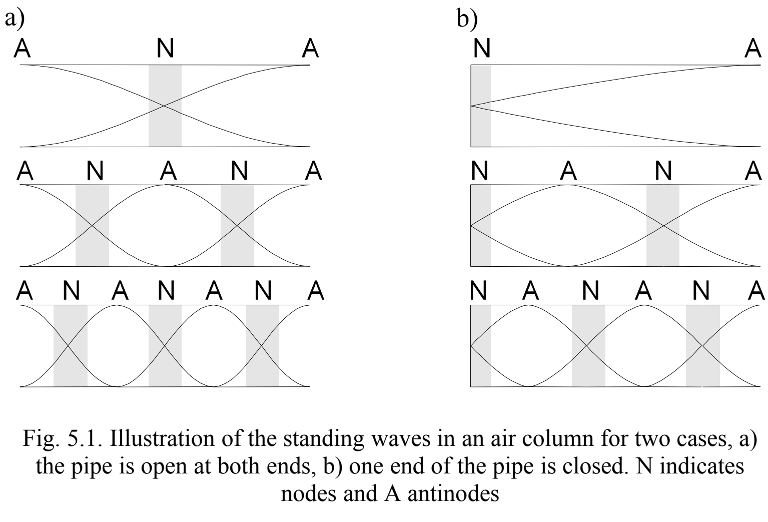

Fig. 5.1. Illustration of the standing waves in an air column for two cases, a) the pipe is open at both ends, b) one end of the pipe is closed. N indicates nodes and A antinodes

The distance between adjacent antinodes is always equal to $\frac{1}{2}\lambda$, one half of the wavelength. For each normal mode the length of the column L must be an integer number of $\frac{1}{2}\lambda$ and the possible wavelengths λ_n are

$$\lambda_n = \frac{2L}{n} \qquad (n = 1, 2, 3, ...) \tag{5.1}$$

If a pipe has one open end and the other end is closed, the modes of vibrations will have an antinode and a node respectively as shown in figure 5.1b. The distance between a node and the adjacent antinode is equal to $\frac{1}{4}\lambda$; one quarter of wavelength. Now for each normal mode the length of the column L must be an odd integer number of $\frac{1}{4}\lambda$ and here the wavelengths are

$$\lambda_n = \frac{4L}{n} \qquad (n = 1, 3, 5, ...) \tag{5.2}$$

The normal mode frequencies for a pipe for both above cases can be calculated from

$$v = \lambda_n f_n \tag{5.3}$$

where v is the speed of sound.

2. The speed of sound in a gas is expressed in terms of its density, ρ, and bulk modulus, B,

$$v = \sqrt{\frac{B}{\rho}} \tag{5.4}$$

The bulk modulus B is defined as

$$B = -\frac{\Delta p}{\frac{\Delta V}{V_0}} \tag{5.5}$$

the ratio of a small change of gas pressure, Δp, to the resulting fractional change in volume $\frac{\Delta V}{V_o}$. B is a positive quantity and the minus sign in (5.5) comes from the fact that an increase in pressure causes a decrease in volume. Thermal conductivity of gases is small and during propagation of sound waves the pressure of the gas undergoes changes which are adiabatic. Hence to obtain B from (5.5) the equation for an adiabatic process is used

$$pV^{\kappa} = \text{const} \tag{5.6}$$

κ is the ratio of the molar heat capacity at constant pressure to that at constant volume.
Taking the derivative of (5.6)

$$B = -V\frac{dp}{dV} = \kappa p \tag{5.7}$$

and from (5.4) the velocity of sound is

$$v = \sqrt{\frac{\kappa p}{\rho}} \tag{5.8}$$

Finally, using for the density ρ of the ideal gas

$$\rho = \frac{pM}{RT} \tag{5.9}$$

it is found that

$$v = \sqrt{\frac{\kappa RT}{M}} \tag{5.10}$$

Here R is the gas constant, M is the molecular mass and T is the absolute temperature. Because κ, R and M are constant, the velocity of the sound wave is proportional to the square root of the absolute temperature of the gas.

APPARATUS AND METHODS

The measurements are carried out using the apparatus shown in figure 5.2. A glass tube about 1 m long is closed at one end with a piston. A small

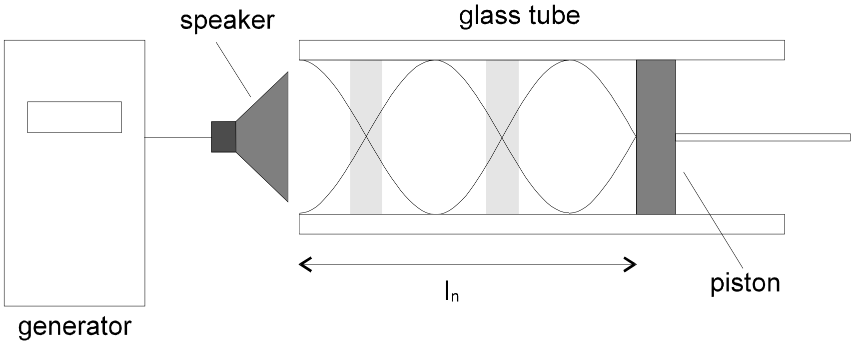

Fig. 5.2. Schematic diagram of the apparatus used to measure the velocity of sound in air

loudspeaker connected to an audio frequency generator is placed at the other end of the tube. The loudspeaker is a source of normal mode vibrations of the air column of length l_n. In the experiment the length of the air column is changed by moving the piston slowly away from the loudspeaker and the lengths l_n are noted for which clear resonances of the air are obtained. The resonances appear at length l_n equal to (see (5.2))

$$l_n = n\frac{\lambda}{4} \qquad (n = 1, 3, 5, ...) \tag{5.11}$$

The wavelength λ is obtained from the lengths l_{n+2} and l_n corresponding to two adjacent resonances

$$\lambda = 2\left(l_{n+2} - l_n\right) \tag{5.12}$$

The velocity of sound for a given frequency f of the generator is next calculated from

$$v = \lambda f \tag{5.13}$$

MEASUREMENTS

1. Find the wavelength λ for 5 different values of frequency in the range 2000-5000Hz. For a given frequency calculate the mean value of the wavelength from the results obtained for all pairs of adjacent resonances. Find its standard deviation (see (6) for the expression used to calculate the standard deviation).

2. Calculate the velocity of sound for each value of frequency. Answer the question whether the velocity is a function of frequency.
3. Calculate the ratio of the molar heat capacity at constant pressure to that at constant volume κ for air from (5.10) taking R=8.315 J/molK and M=28.8 $\times 10^{-3}$ kg/mol.

ANALYSIS OF ERRORS

The error in the measurements of velocity of sound is determined as the standard deviation

$$\sigma_v = \sqrt{(f\,\sigma_\lambda)^2 + (\lambda\,\sigma_f)^2} \tag{5.14}$$

where σ_λ and σ_f are standard deviations in the measurement of λ and f respectively. The standard deviation in the measurement of frequency σ_f = 3Hz.

QUESTIONS

1. Find the normal mode frequencies for a pipe closed at both ends.
2. Discuss the variation of gas pressure at the position of nodes and antinodes of a standing wave.
3. How much different from the measured value would the velocity of sound in the gas be if the pressure in a propagating wave varies according to the isothermal process?
4. What is the difference between the velocity of sound in air in winter at -25 °C and in summer at 35°C?