## Experiment 6

## AN INVESTIGATION OF THE PRESSURE DEPENDENCE OF THE BOILING-POINT OF WATER

The aim of the experiment is to investigate the pressure dependence of the boiling-point of water in the pressure range below normal atmospheric pressure. From the obtained dependence the latent heat of vaporization is determined.

## INTRODUCTION

A liquid is boiling when bubbles of vapour form intensively within the bulk of the liquid. A bubble of radius r can form at a depth h below the surface (figure 6.1) when its vapour pressure p is

$$
\begin{equation*}
\mathrm{p}=\mathrm{p}_{\mathrm{ex}}+\mathrm{h} \rho \mathrm{~g}+\frac{2 \sigma}{\mathrm{r}} \tag{6.1}
\end{equation*}
$$



Fig. 6.1. Formation of a bubble of vapour of diameter $2 r$ at a depth $h$ below the surface of the liquid. $\mathrm{p}_{\mathrm{ex}}$ is the external pressure
$\mathrm{p}_{\mathrm{ex}}$ is the external pressure on the surface, $\mathrm{h} \rho \mathrm{g}$ is the hydrostatic pressure of the liquid and $2 \sigma / \mathrm{r}$ is an additional pressure on the bubble from its surface tension $\sigma .2 \sigma / \mathrm{r}$ is much smaller than $\mathrm{p}_{\mathrm{ex}}$ for a vapour bubble formed at a creation center (small particles in the liquid) and if hydrostatic pressure is also neglected in (6.1) it is seen that $\mathrm{p}=\mathrm{p}_{\mathrm{ex}}$. A liquid boils when its saturated vapour pressure is equal to the external pressure. However the vapour pressure depends on its
temperature. The boiling point of a liquid is then that temperature at which the vapour pressure of the liquid is equal to the external pressure.

The effect of the temperature T on vapour pressure p is described by the Clausius-Clapeyron equation

$$
\begin{equation*}
\frac{\mathrm{dp}}{\mathrm{dT}}=\frac{\lambda}{\mathrm{T}} \frac{1}{\mathrm{~V}-\mathrm{V}_{\mathrm{L}}} \tag{6.2}
\end{equation*}
$$

where $\lambda$ is the latent heat of vaporization (per mole), $\mathrm{V}_{\mathrm{L}}$ is the volume occupied by the liquid (one mole) and V is the volume of one mole of the vapour. In the integration of (6.2) it is assumed that $\mathrm{V}_{\mathrm{L}} \ll \mathrm{V}$ and that $\lambda$ is independent of the temperature which is a good approximation based on modern measurements of the heat of vaporization. If it is further assumed that the vapour behaves as an ideal gas

$$
\begin{equation*}
\mathrm{V}=\frac{\mathrm{RT}}{\mathrm{p}} \tag{6.3}
\end{equation*}
$$

equation (6.2) becomes

$$
\begin{equation*}
\frac{\mathrm{dp}}{\mathrm{dT}}=\frac{\lambda}{\mathrm{R}} \frac{\mathrm{p}}{\mathrm{~T}^{2}} \tag{6.4}
\end{equation*}
$$

From the integration

$$
\begin{equation*}
\ln \mathrm{p}=-\frac{\lambda}{\mathrm{R}} \frac{1}{\mathrm{~T}}+\mathrm{C} \tag{6.5}
\end{equation*}
$$

and further

$$
\begin{equation*}
\mathrm{p}=\mathrm{C} \mathrm{e}^{-\frac{\lambda}{\mathrm{R}} \frac{1}{\mathrm{~T}}} \tag{6.6}
\end{equation*}
$$

where C is a constant, which can be found from (6.6) knowing that the liquid boils at $\mathrm{T}_{0}$ under standard pressure $\mathrm{p}_{0}$.

## APPARATUS AND METHOD

The apparatus used for the measurements of the boiling-point of water is shown in figure 6.2. The flask F contains water and it is connected through a
condenser C to a vessel V which is an air reservoir. The pressure of air in the


Fig. 6.2. Apparatus used for the measurements of the boiling-point of water. F is a flask containing water, C is a condenser, T is a thermometer, M is a manometer and K is a valve
flask F is adjusted by using a vacuum pump to decrease the pressure or valve K to increase the pressure by letting small amounts of air into the reservoir. The pressure in the apparatus is recorded by a manometer M . The water in the flask is steadily heated and at a given pressure the temperature of the boiling point is recorded by the thermometer T. By changing the pressure in the reservoir, the boiling point of water is obtained for different values of pressure below normal atmospheric pressure.

Typical results obtained in the experiment are shown in figure 6.3 where pressure p using a logarithmic scale is plotted as a function of the inverse temperature $\frac{1}{\mathbf{T}}$. A straight line is drawn between the experimental points according to (6.5) which predicts a linear dependence between $\ln \mathrm{p}$ and $\frac{1}{\mathrm{~T}}$. The slope of the graph is $-\frac{\lambda}{R}$ and by measuring the slope of the line in figure 6.3 the latent heat of vaporization per mole, $\lambda$, is found where

$$
\begin{equation*}
\lambda=-\mathrm{R} \frac{\ln \mathrm{p}_{2}-\ln \mathrm{p}_{1}}{\frac{1}{\mathrm{~T}_{2}}-\frac{1}{\mathrm{~T}_{1}}} \tag{6.7}
\end{equation*}
$$

Here $\mathrm{p}_{2}, \mathrm{p}_{1}$ and $\frac{1}{\mathrm{~T}_{2}}, \frac{1}{\mathrm{~T}_{1}}$ are the p and $\frac{\mathbf{1}}{\mathbf{T}}$ coordinates respectively of two points lying on the straight line.


Fig. 6.3. A graph of pressure p as a function of the inverse temperature $\frac{1}{\mathrm{~T}}$. A logarithmic scale is used for pressure p

## MEASUREMENTS

1. Measure the boiling point of water at 10 different values of the pressure in the range below normal atmospheric pressure.
2. Plot a graph of the obtained $p=f(T)$ dependence using a linear scale and a graph of the $\ln p=f(1 / T)$ dependence using a logarithmic scale.
3. Find the latent heat of vaporization of water using equation (6.7). Take $\mathrm{R}=8.315 \mathrm{~J} / \mathrm{mol} \mathrm{K}$. Compare the obtained value with that given in table 7 .

## ANALYSIS OF ERRORS

During the experiment the errors in the measurements of temperature, $\Delta \mathrm{T}$, and pressure, $\Delta \mathrm{p}$, have to be established. These errors are next used in the graphs to draw the error bars. The error in the obtained latent heat of vaporization, $\Delta \lambda$, can be determined using a graphical method.

## QUESTIONS

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1. Recent measurements show that the latent heat of vaporization of water changes with temperature at a rate of $2.5 \mathrm{~kJ} / \mathrm{kg} \mathrm{K}$. How big is the error in (6.6) that results from taking $\lambda$ to be constant over the temperature range of the experiment?
2. At the height of Everest, 9000 m , the atmospheric pressure is equal to 280 mmHg . What is the boiling point of water at that height?
3. Give a definition of pressure and state the units used to measure it. What does the pressure in a fluid depend on?
