Experiment 10

MEASUREMENT OF CAPACITANCE USING AN ALTERNATING CURRENT BRIDGE

In this experiment capacitance is measured with the help of a Wheatstone bridge used for alternating current. Also the series and parallel connections of the capacitors are studied.

INTRODUCTION

A capacitor is formed by two conductors which are separated by an insulator. When the capacitor is charged and the potential difference between the conductors is V, the conductors have charges with equal magnitude Q and opposite sign. The capacitance C of a capacitor is defined as the ratio of the charge to the potential difference

$$C = \frac{Q}{V} \tag{10.1}$$

The SI unit of capacitance is the farad (F).

A capacitor stores electric potential energy which is equal to the energy required to charge the capacitor. The work dW required to transfer an element of charge dq between the conductors is

$$dW = V\,dq = \frac{q}{C}\,dq \tag{10.2}$$

The total work W needed to charge the capacitor to a final charge Q is equal to the potential energy U of a charged capacitor. From the integration of (10.2)

$$U = W = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 \tag{10.3}$$

The capacitors in different applications are connected in series (figure 10.1a) or in parallel (figure 10.1b) to obtain particular values of final capacitance. In

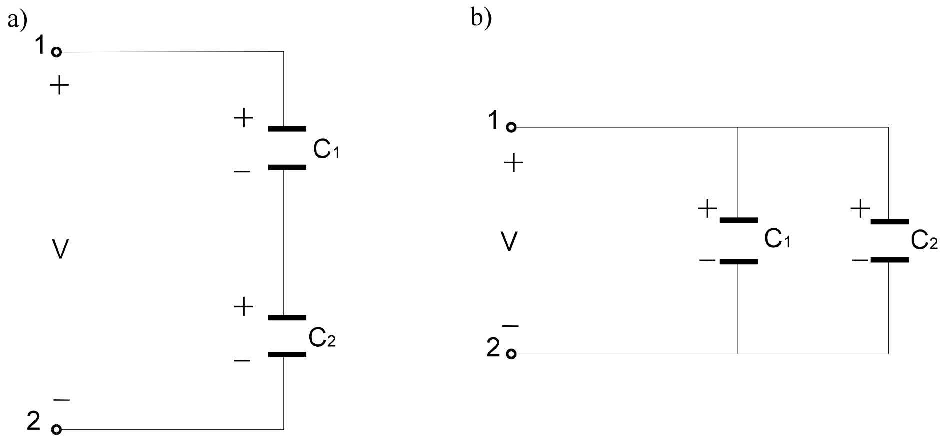

Fig. 10.1. Capacitors C_1 and C_2 connected a) in series, b) in parallel. V is the potential difference between points 1 and 2

a series connection the reciprocal of the equivalent (total) capacitance is equal to the sum of the reciprocals of individual capacitances

$$\frac{1}{C_e} = \sum_i \frac{1}{C_i} \tag{10.4}$$

In a parallel connection the equivalent capacitance is equal to the sum of the individual capacitances

$$C_e = \sum_i C_i \tag{10.5}$$

Capacitors are most commonly used in circuits conducting alternating (a.c.) current which varies sinusoidally with time. For a capacitor having instantaneous charge q

$$i = \frac{dq}{dt} = I_0 \cos \omega t \tag{10.6}$$

Integrating (10.6) and using (10.1) the capacitor voltage

$$V = \frac{I_0}{\omega C} \sin \omega t = \frac{I_0}{\omega C} \cos\left(\omega t - \frac{\pi}{2}\right) \tag{10.7}$$

The maximum voltage is equal to

$$V_0 = \frac{I_0}{\omega C} = I_0 X_C \tag{10.8}$$

where the quantity X_C called the capacitive reactance of the capacitor is

$$X_C = \frac{1}{\omega C} \tag{10.9}$$

X_C is an analogous quantity to the resistance R of a resistor for which V = IR.

APPARATUS AND METHOD

Capacitance is measured using an electrical bridge circuit shown schematically in figure 10.2.

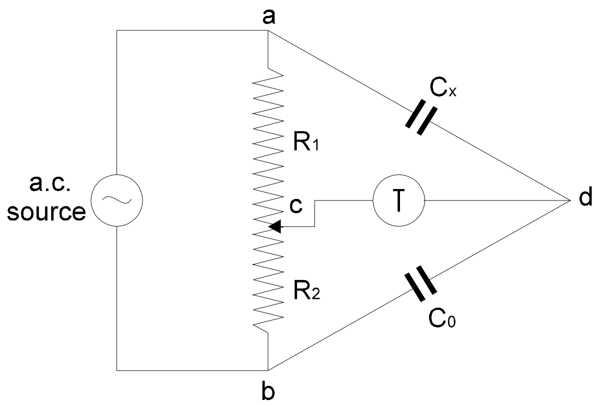

Fig 10.2. Diagram of an electric bridge circuit used in the measurement of the capacitance

An alternating current source of audio frequency is connected to the bridge circuit. C_0 is a condenser with known capacitance and the C_x capacitance is to be measured. The R_1 and R_2 resistances are varied by moving a slider along a resistive wire connected between points a,b until an impedance balance is obtained for the bridge. This is indicated by zero or minimum sound in the earphone detector T. The voltage difference between points c,d is then zero and for the impedances the balance equation holds

$$\frac{R_1}{\frac{1}{\omega C_x}} = \frac{R_2}{\frac{1}{\omega C_0}} \tag{10.10}$$

Here the capacitive reactances have been used. The ratio of resistances equal to

$$\frac{R_2}{R_1} = \frac{C_x}{C_0} \tag{10.11}$$

is obtained in the measurements of the lengths l_1 and l_2 of the sections of the resistive wire which correspond to R_1 and R_2 respectively. The measured capacitance is then, from (10.11)

$$C_x = C_0 \frac{l_2}{l_1} \tag{10.12}$$

MEASUREMENTS

1. Find the capacitance of the capacitors provided. Repeat the measurements 3 times for each capacitor. The highest sensitivity in the measurements is obtained for values of l_1 and l_2 that are close to each other.
2. Find the total capacitance of the capacitors connected a) in series, b) in parallel. Check whether the results of these measurements are in agreement with the calculations made using results from 1.

ANALYSIS OF ERRORS

The error in the measurements of C_x is calculated from

$$\Delta C_x = C_x \left(\frac{\Delta l_1}{l_1} + \frac{\Delta l_2}{l_2} + \frac{\Delta C_0}{C_0} \right) \tag{10.13}$$

where Δl_1, Δl_2 are the errors in the measurements of the lengths l_1, l_2 and ΔC_0 is the error in C_0. These errors have to be determined in the procedure of balancing the bridge.

QUESTIONS

1. Show that the balance equation for the bridge circuit shown in figure 10.2 when the two capacitors are replaced by resistors R_x and R_0 respectively is

$$\frac{R_1}{R_x} = \frac{R_2}{R_0}$$

2. Explain what does the capacitance of a capacitor depend on.
3. Obtain equation (10.4) for a series connection of capacitors.
4. Obtain equation (10.5) for a parallel connection of capacitors.