

Experiment 11

AN INVESTIGATION OF AN ALTERNATING CURRENT CIRCUIT WITH INDUCTANCE, CAPACITANCE AND RESISTANCE

In this experiment alternating current circuits with inductance, capacitance and resistance are studied and the currents in the circuits are measured as a function of the source frequency. From these measurements the inductance of a magnetic coil is determined.

INTRODUCTION

A magnetic coil with self-inductance L and assumed zero resistance is connected to an alternating current (a.c.) source as shown in figure 11.1a.

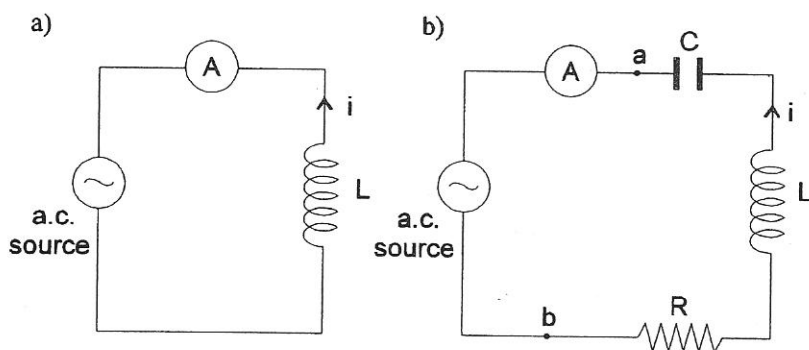


Fig 11.1. Diagrams of electrical alternating current circuits containing
a) a coil, and b) a resistor, a coil and a capacitor

The current in the circuit changes sinusoidally with time

$$i = I_0 \cos \omega t \quad (11.1)$$

and the potential difference across the coil is

$$V_L = L \frac{di}{dt} = -I_0 \omega L \sin \omega t = I_0 \omega L \cos \left(\omega t + \frac{\pi}{2} \right) \quad (11.2)$$

I_0 is the current amplitude. The voltage across the coil and the current are out of phase and the voltage peaks a quarter of the period earlier than the current. The voltage leads the current by the phase angle difference equal to $\frac{\pi}{2}$.

The voltage amplitude is

$$V_0^L = I_0 \omega L \quad (11.3)$$

The inductive reactance X_L of a coil is defined as

$$X_L = \frac{V_0^L}{I_0} = \omega L \quad (11.4)$$

and it has the same units as resistance. The inductive reactance of a coil is directly proportional to its inductance L and to the angular frequency ω .

When a capacitor with capacitance C is connected to the same a.c. source of figure 11.1a it has been shown in Experiment 10 that the voltage across the capacitor is

$$V_c = \frac{I_0}{\omega C} \cos\left(\omega t - \frac{\pi}{2}\right) \quad (11.5)$$

It is seen by comparing (11.1) and (11.5) that the capacitor voltage and current are not in phase and the voltage peaks a quarter of the period after the current.

The voltage lags the current by the phase angle of $-\frac{\pi}{2}$.

The capacitive reactance of the capacitor is

$$X_c = \frac{V_0^c}{I_0} = \frac{1}{\omega C} \quad (11.6)$$

It is inversely proportional to the capacitance and the angular frequency.

A circuit containing a resistor, a coil and a capacitor connected in series is shown in figure 11.1b. The potential difference between terminals a and b is equal to the sum of the potential differences across each component R , L , and C . It can be written in a way similar to equations (11.2) and (11.5) as

$$V = V_0 \cos(\omega t + \Phi) \quad (11.7)$$

where the voltage amplitude V_0 is

$$V_0 = I_0 Z \quad (11.8)$$

Φ is the phase angle and is given by

$$\tan \Phi = \frac{X_L - X_C}{R} = \frac{1}{R} \left(\omega L - \frac{1}{\omega C} \right) \quad (11.9)$$

Equation (11.8) defines the impedance Z of the circuit which is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2} \quad (11.10)$$

The impedance is a function of R , L and C and also of the angular frequency ω . When the circuit is connected to a voltage source with constant amplitude the current amplitude will vary with frequency according to equation (11.8). The maximum value of the current occurs at the frequency ω_0 at which the impedance Z takes its minimum value. ω_0 is called the resonance angular frequency and at this frequency a resonance occurs in the circuit. The impedance Z is minimum at a frequency at which the inductive reactance is equal to the capacitive reactance, $X_L = X_C$ (see 11.10). Then the frequency is

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad (11.11)$$

APPARATUS AND METHOD

The inductance of a magnetic coil is measured using two methods. In the first, the resonance method, the current in the circuit containing resistance, inductance and capacitance (figure 11.1b) is measured as a function of frequency f . Figure 11.2 shows a graph of the resonance curve obtained in typical measurements. The resonance frequency f_0 is determined from the graph and the inductance is calculated from

$$L = \frac{1}{4\pi^2 f_0^2 C} \quad (11.12)$$

where C is the capacitance of a capacitor in the circuit.

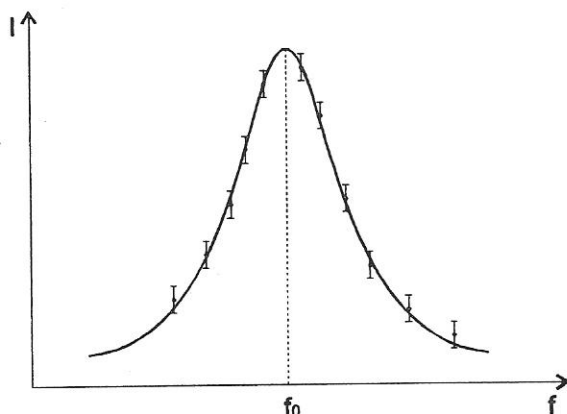


Fig. 11.2. A graph showing current I in an alternating current circuit as a function of frequency f . f_0 is the resonance frequency

In the second method the capacitor is removed from the circuit leaving just the inductance and resistance and again the current is recorded as a function of frequency f . Reading the voltage between terminals a and b the impedance Z is next calculated using (11.8). For the present circuit

$$Z^2 = R^2 + 4\pi^2 f^2 L^2 \quad (11.13)$$

A graph of Z^2 against f^2 yields a straight line as shown in figure 11.3. The straight line intercepts the f^2 axis at point $-f_x^2$ and now the inductance can be

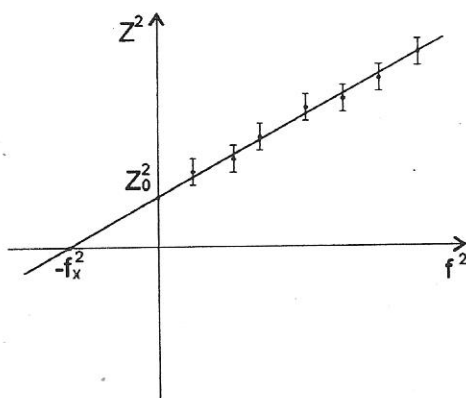


Fig. 11.3. A graph showing the square of the impedance, Z^2 , of an alternating current circuit as a function of the square of the frequency, f^2 . $-f_x^2$ is the point of intercept of the f^2 axis by the straight line

calculated from

$$L = \frac{R}{2\pi} \frac{1}{\sqrt{f_x^2}} \quad (11.14)$$

R is the resistance in the circuit which also includes the resistance of the coil whose inductance is being determined.

MEASUREMENTS

1. Obtain the resonance curves in the frequency region 200-2000Hz for two different coils connecting the circuit of figure 11.1b. Find the resonance frequencies and calculate the inductances using (11.12).
2. Remove the capacitor from the circuit and measure the frequency dependence of the current for the same coils in the frequency region 1000-5000Hz. From the graph of Z^2 against f^2 find f_x^2 and calculate the inductances of the coils using (11.14). The value of resistance $R = \sqrt{Z_0^2}$, where Z_0^2 is the coordinate of the point of interception of the Z^2 axis by the straight line.

ANALYSIS OF ERRORS

In the first, resonance method the error in the determination of the inductance is calculated from

$$\Delta L = L \left(2 \frac{\Delta f}{f_0} + \frac{\Delta C}{C} \right) \quad (11.15)$$

where Δf is the error in the determination of the resonance frequency and ΔC is the error in the value of capacitance which is equal to $\Delta C = 0.05C$.

In the second method the error in the inductance is

$$\Delta L = L \left(\frac{\Delta R}{R} + \frac{1}{2} \frac{\Delta f_x^2}{f_x^2} \right) \quad (11.16)$$

Here ΔR is the error in the value of the resistance R and Δf_x^2 is the error in the determination of f_x^2 . Both errors are found graphically from the plot (figure 11.3).

The error in the square of the impedance, Z^2 , to be indicated in figure 11.3, is calculated from

$$\Delta(Z^2) = 2 Z \Delta Z \quad (11.17)$$

$$\Delta Z = \left| \frac{1}{I_0} \right| \Delta V_0 + \left| \frac{V_0}{I_0^2} \right| \Delta I_0 \quad (11.18)$$

where ΔV_0 and ΔI_0 are errors in the value of the voltage V_0 and the current I_0 respectively.

QUESTIONS

1. Draw graphs of voltage and current as functions of time for circuits studied in the experiment and show the phase angle difference.
2. How would the shape of the resonance curve change when the resistance R in the circuit is decreased by a factor of two?
3. Variations of the voltage and current in the a.c. circuits are often represented by vectors called phasors which rotate counterclockwise with constant angular velocity ω . Draw a phasor diagram for the circuit shown in figure 11.1a.
4. Calculate the average power in the circuit of figure 11.1a.