Experiment 12

DETERMINATION OF THE HORIZONTAL COMPONENT OF THE EARTH'S MAGNETIC FIELD

The horizontal component of Earth's magnetic field is measured using a tangent galvanometer.

INTRODUCTION

A magnetic field exerts a force $\mathbf{F}$ on a charge moving in the field which is proportional to the value of the charge q and its velocity $\mathbf{v}$

$$\mathbf{F} = q\, \mathbf{v} \times \mathbf{B} \tag{12.1}$$

$\mathbf{B}$ is the magnetic field. The direction of the force $\mathbf{F}$ is perpendicular to the plane containing both vectors $\mathbf{v}$ and $\mathbf{B}$ as shown in figure 12.1 for a positive charge and two opposite directions of the velocity. For the negative charge the directions of the forces are opposite to that in the figure.

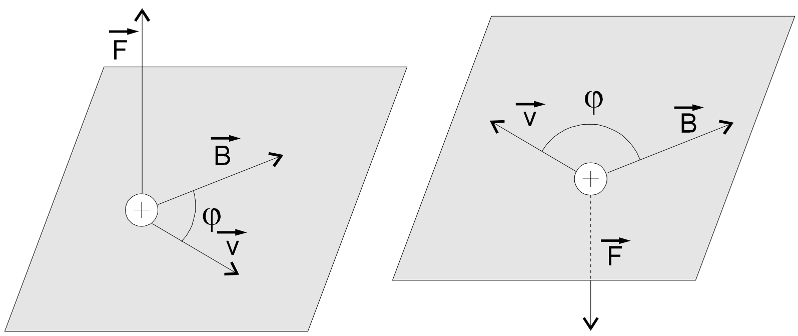

Fig. 12.1. Force $\mathbf{F}$ acting on a positive charge moving with velocity $\mathbf{v}$ in amagnetic field $\mathbf{B}$ for two opposite directions of the velocity

The magnitude of the force $\mathbf{F}$ is given by

$$F = |q|\, v\, B \sin\varphi \tag{12.2}$$

where φ is the angle between the velocity and the magnetic field (figure 12.1). From equation (12.2) the units of B are $1\frac{Ns}{Cm} = 1\frac{N}{Am}$ which is called tesla (1T).

Magnetic fields are produced by currents in conductors. The law of Biot and Savart states that the magnetic field $\mathbf{dB}$ due to a short segment $\mathbf{dl}$ of a conductor carrying current i is given by

$$\vec{dB} = \frac{\mu_0}{4\pi}\frac{i\,dl \times r}{r^3} \tag{12.3}$$

where $\mathbf{dB}$ is at a point whose position is described by vector $\mathbf{r}$ as illustrated in figure 12.2. The direction of $\mathbf{dB}$ is determined from the vector product of $\mathbf{dl}$ and $\mathbf{r}$. $\frac{\mu_0}{4\pi}$ is a proportionality constant and is equal to $10^{-7}\,\frac{Tm}{A}$. μ_0 is the permeability constant. The total magnetic field $\mathbf{B}$ due to the current in a complete

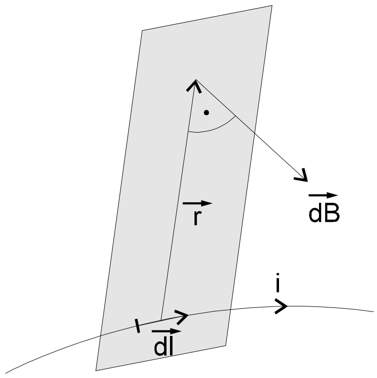

Fig 12.2. Magnetic field $\mathbf{dB}$ due to a segment $\mathbf{dl}$ of a conductor carrying current i

conductor is obtained from the integration of (12.3) over the whole length of the conductor

$$\vec{B} = \int \vec{dB} = \frac{\mu_0}{4\pi}\int\frac{i\,dl \times r}{r^3} \tag{12.4}$$

Using (12.4) the magnetic field produced by conductors of different shape can be calculated. For a straight conductor of length much greater than the distance r of the point from the conductor the magnetic field is

$$B = \frac{\mu_0}{2\pi}\frac{i}{r} \tag{12.5}$$

The field has an axial symmetry. $\mathbf{B}$ has the same value on a circle centered on the conductor and its direction is tangential to that circle (figure 12.3a).

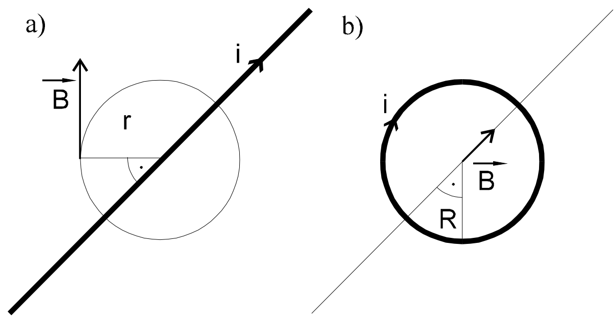

Fig. 12.3. Magnetic field $\mathbf{B}$ due to a) straight conductor carrying current i and b) circular conductor carrying current i

A circular conductor with radius R carrying a current i produces a magnetic field which at the center of the circle is

$$B = \frac{\mu_0}{2}\frac{i}{R} \tag{12.6}$$

$\mathbf{B}$ is perpendicular to the surface containing the circle (figure 12.3b)

APPARATUS AND METHOD

The measurements are made using a tangent galvanometer shown in figure 12.4a. It contains a circular magnetic coil C and a magnetic needle E whose position is measured using an angular scale A. The needle is deflected by the resultant magnetic field produced by the coil and the magnetic field of the Earth and it points in the direction of the resultant field (figure 12.4b). If the plane of the galvanometer coil is in the meridian (N-S direction) and the deflection of the needle is φ, the horizontal component of the Earth's magnetic field B_E is given by

$$B_E = \frac{B_C}{\tan\varphi} \tag{12.7}$$

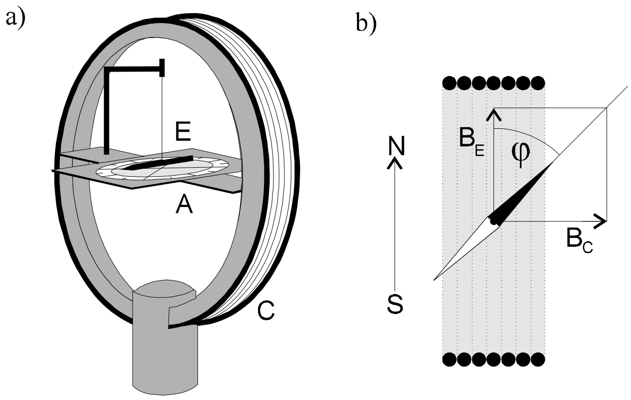

Fig. 12.4. a) Tangent galvanometer used to measure the horizontal component of the Earth's magnetic field. C is the circular magnetic coil, E is a magnetic needle and A is the angular scale. b) Deflection of the magnetic needle in the fields of the magnetic coil, B_C, and the magnetic Earth's field B_E. N - S shows the direction of the meridian

where B_C, the magnetic field of the coil is equal to

$$B_C = \frac{\mu_o n\, i}{2R} \tag{12.8}$$

The tangent galvanometer is connected to a regulated direct current power supply P (figure 12.4a) via a reversing key R which allows the direction of the current in the coil and thus the direction of the magnetic field B to be reversed. The deflection angle is measured for both directions of the current giving deflection of the needle to the left, φ_L, and to the right, φ_R, in respect to the plane of the coil. The value of the deflection angle φ for a given value of B_C is taken as the average

$$\varphi = \frac{\varphi_L + \varphi_R}{2} \tag{12.9}$$

The value of B_C is changed in the experiment by adjusting the current i or by adding an appropriate number of turns to the coil (see equation 12.8).

The horizontal component of the field B_E is calculated from (12.7) using least-squares fitting to the linear dependence between B_C and tan φ (equation 12.7) as B_E is constant. It is given by

$$B_E = \frac{\sum_{i=1}^{n} \tan\varphi_i \, B_C^i}{\sum_{i=1}^{n} \left(\tan\varphi_i\right)^2} \tag{12.10}$$

where n is the number of measured points. The magnetic field intensity H_E corresponding to B_E is defined by the relation

$$B_E = \mu_0 H_E \tag{12.11}$$

MEASUREMENTS

1. Measure the deflection of the magnetic needle as a function of the current in the coil over the range 0 - 1A for 2, 4 and 6 turns in the magnetic coil. Make the measurements for both directions of the current and use (12.9) to calculate φ. Make sure that the plane of the coil is in the meridian by setting it in the direction of the magnetic needle when the current is zero.
2. Calculate B_E and H_E from (12.10) and (12.11) and draw a plot of tan φ against B_C. On the graph draw a straight line with the slope corresponding to the calculated value of B_E (equation 12.7).

ANALYSIS OF ERRORS

The uncertainty σ_B given here as standard deviation in the obtained value of B_E is calculated from the expression

$$\sigma_B^2 = \frac{1}{n-2}\left(\frac{\sum_{i=1}^{n} \left(B_C^i\right)^2}{\sum_{i=1}^{n} \left(\tan\varphi_i\right)^2} - B_E^2 \right) \tag{12.12}$$

which is derived within the least-squares fitting method.

The errors in the directly measured quantities, deflection angle φ and current i, $\Delta\varphi$ and Δi respectively are determined during their read out. They are next used to find the error in the magnetic field

$$\Delta B_C = B_C\left(\frac{\Delta i}{i} + \frac{\Delta R}{R}\right) \tag{12.13}$$

and the error in the tan φ which is given by

$$\Delta(\tan\varphi) = \frac{1}{\cos^2\varphi}\Delta\varphi \tag{12.14}$$

ΔR is the error in the radius of the coil. These errors, ΔB_C and $\Delta(\tan\varphi)$ are next used to draw error bars in the graph of tan φ as a function of B_E.

QUESTIONS

1. Give the definition of the magnetic field vector $\mathbf{B}$ and compare it with the definition of electric field vector $\mathbf{E}$.
2. Describe the motion of a charged particle in a uniform magnetic field.
3. Describe how a magnetic field effects a conductor carrying current.
4. Calculate the magnetic field B at the centre of a circular orbit of radius r produced by an electron moving with velocity v.