Experiment 16

DETERMINATION OF THE WAVELENGTH DEPENDENCE OF THE REFRACTIVE INDEX OFAGLASS PRISM

In this experiment the refractive index of a glass prism is determined for different values of the wavelength of light in the visible region 0.4 - 0.65 μm. Cauchy's formula for the refractive index is also investigated.

INTRODUCTION

The refractive index of an optical material depends on the wavelength of the light. This dependence is called dispersion and is a result of the fact that the speed of light in a material is different for different wavelengths. Light of longer wavelength (e.g. red light) usually has a greater speed than light of shorter wavelength (e.g. violet light).

The refractive index n (see Experiment 15 for definition) usually decreases with increasing wavelength λ and this dependence is given by Cauchy's formula

$$n = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + \ldots \tag{16.1}$$

where A, B and C are empirically determined constants for a particular material. The C term in (16.1) is often very small and is usually neglected.

Cauchy's formula holds for so called normal dispersion which exists in colourless transparent materials such as glass or water. However if there is absorption of light at a given wavelength the Cauchy's formula does not apply and this is called anomalous dispersion.

White light which is a superposition of waves having different wavelengths in the visible region becomes dispersed when passing from one material to another as happens, for example, in a prism. Figure 16.1 shows how the monochromatic light passes through a prism with an apex angle A. An incoming ray is refracted at both boundary surfaces and undergoes deviation with an angle of deviation δ equal to

$$\delta = \left(\Theta_i - \Theta_r\right) + \left(\Theta_i' - \Theta_r'\right) = \left(\Theta_i + \Theta_i'\right) - \left(\Theta_r + \Theta_r'\right) \tag{16.2}$$

Also the apex angle A is

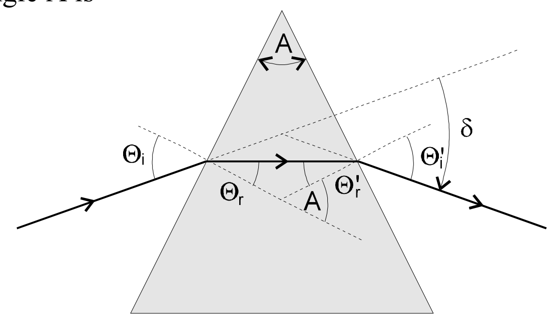

Fig. 16.1. Refraction of monochromatic light in a glass prism with an apex angle A. δ is an angle of deviation of the incident ray

$$A = \Theta_r + \Theta_r' \tag{16.3}$$

The angle of deviation is a minimum when the ray passes symmetrically through the prism, $\theta_i = \theta_i'$ and $\theta_r = \theta_r'$ and from (16.2) and (16.3) it is obtained that

$$\Theta_i = \frac{\delta_{min} + A}{2} \quad , \quad \Theta_r = \frac{A}{2} \tag{16.4}$$

Finally for the minimum deviation, δ_{min}, the refractive index is given by

$$n = \frac{\sin\Theta_i}{\sin\Theta_r} = \frac{\sin\frac{\delta_{min} + A}{2}}{\sin\frac{A}{2}} \tag{16.5}$$

The deviation of monochromatic light produced by a prism depends on the refractive index. For example, violet light deviates more than red light because it has a larger value of n.

APPARATUS AND METHOD

The optical spectrometer used to measure the refractive index is shown in figure 16.2. The spectrometer consists of a collimator which serves to provide a parallel beam of light from a source, a prism to disperse the light and a telescope to record a spectrum of the dispersed light. The position of the telescope, which can rotate about the common axis of the spectrometer, is read from an angular scale and is used to measure the deviation of the light. The position of the telescope is adjusted to observe the image of a given line in the spectrum using the cross wires of the telescope.

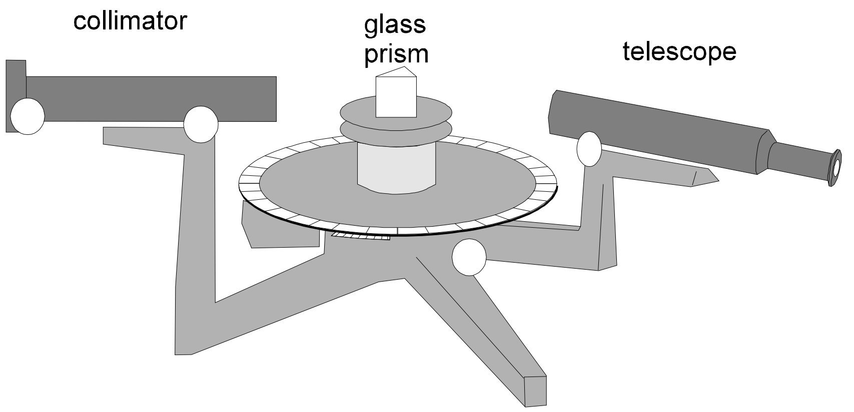

Fig. 16.2. Optical spectrometer used to measure the refractive index of the glass prism

To find the refractive index from (16.5) for a given wavelength of the light, the minimum angle of deviation for that line in the dispersed spectrum and the apex angle of the prism have to be measured. The apex angle of the prism is obtained using the following procedure. The prism is set in a position with the refracting edge pointing towards the collimator. The telescope is moved to find the images of the collimator slit formed when light is reflected from two side

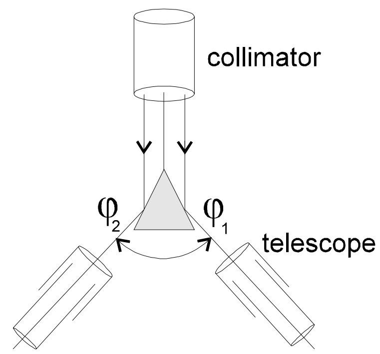

Fig. 16.3. Reflection of light from two side surfaces of the glass prism in the measurements of the apex angle of the prism A. φ_1 and φ_2 indicate the two position of the telescope

surfaces of the prism (figure 16.3). The angle between the two position of the telescope φ_1 and φ_2 is twice the apex angle of the prism and hence

$$A = \frac{1}{2}\left|\varphi_1 - \varphi_2\right| \tag{16.6}$$

To find the minimum angle of deviation the orientation of the prism is adjusted to obtain the dispersed spectrum of the source of light. Now for a given line in the spectrum (light of a certain colour) the prism is rotated while the image of the line is followed with the telescope up to the position of the minimum deviation φ_m. The angle of deviation is

$$\delta_{min} = \left|\varphi_m - \varphi_0\right| \tag{16.7}$$

where φ_0 is the position of the telescope to observe the incident light when the prism is removed from the table. This procedure is repeated for each line in the spectrum for which wavelengths are given in table 6. Next the refractive index n is calculated from (16.5) for all the wavelengths measured and a graph is drawn of n against $\frac{1}{\lambda^2}$ which according to the Cauchy's formula (equation 16.1. with neglected C term) should yield a straight line (figure 16.4). The constants

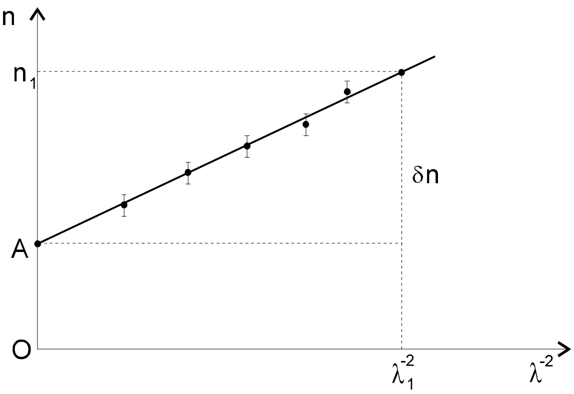

Fig. 16.4. A graph of the refractive index, n, as a function of the inverse square of the wavelength, λ^{-2}. Extrapolation of the straight line intercepts the n axis at point (0,A)

A and B in (16.1) can be found from the graph; A from the extrapolation of the line to intercept the n axis and B from the slope of the straight line, since

$$B = \frac{\delta n}{\lambda_1^{-2}} = \frac{n_1 - A}{\lambda_1^{-2}} \qquad (16.8)$$

Here n_1 and λ_1^{-2} are the coordinates of a point lying on the straight line.

MEASUREMENTS

1. Find the apex angle of the glass prism. Repeat the measurements 7 times and calculate the mean value with its standard deviation.
2. Measure the wavelength dependence of the refractive index in the region of visible light.
3. Draw a graph of n against $\frac{1}{\lambda^2}$ and find the A and B constants in the Cauchy's formula.

ANALYSIS OF ERRORS

The error in the measurement of the refractive index is calculated from the formula

$$\Delta n = \frac{1}{2}\left|\frac{\cos\frac{\delta_{min} + A}{2}}{\sin\frac{A}{2}}\right|\Delta\delta + \frac{1}{2}\left|\frac{\sin\frac{\delta_{min}}{2}}{\sin\frac{A}{2}}\right|\Delta A \qquad (16.9)$$

where $\Delta\delta$ is the error in the minimum deviation angle measurement which should be determined during the experiment. The error ΔA to be used for the apex angle is taken to be 3 σ_A where σ_A is the standard deviation of A. The values of Δn are used to draw the error bars in the plot of n against $\frac{1}{\lambda^2}$.

QUESTIONS

1. Show that equation (16.6) holds for the measurement procedure of the apex angle of the prism.
2. How does the wavelength, frequency and velocity of the light waves change on passage from one material to another?

3. Describe the refraction of light in water drops to form a rainbow.

4. Light travels from point A to point B when it is reflected from a surface at distance x. Show that the time required to reach B is a minimum when the angle of incidence is equal to the angle of reflection (Fermat's principle of least time).