Experiment 17

MEASUREMENT OF THE RADIUS OF CURVATURE OF A LENS BY THE METHOD OF NEWTON'SRINGS

The effect of interference of monochromatic light in a thin film of air which produces Newton's rings is used to determine the radius of a glass lens.

INTRODUCTION

Interference of two or more waves occurs when the waves overlap in space. The resultant displacement at any point of space and any moment of time is described by the principle of linear superposition which says that the resultant displacement is equal to the sum of the instantaneous displacements that would be produced by the individual waves independently. Here the displacement term is used in a general sense. For example for mechanical waves it applies to the mechanical displacement while for electromagnetic waves it means the amplitude of electric or magnetic fields.

The interference patterns are clearly observed in the case of waves for which there is a constant phase difference. Such waves are called coherent. Two monochromatic waves which are coherent and additionally which are in phase would interfere in space constructively or destructively depending on the difference in paths between each source and the point of interference, p_1 and p_2. Constructive interference occurs when the path difference is equal to an integral multiple of the wavelength λ

$$\Delta p = p_2 - p_1 = n\lambda \qquad (n = 0, \pm 1, \pm 2, \ldots) \tag{17.1}$$

For two waves with the same amplitudes the resultant amplitude is twice the amplitude of the incoming waves. The destructive interference (cancellation) on the other hand occurs when the path difference is

$$\Delta p = p_2 - p_1 = \left(n + \frac{1}{2}\right)\lambda \qquad (n = 0, \pm 1, \pm 2, \ldots) \tag{17.2}$$

and the resultant amplitude is zero when both waves have the same amplitudes.

Interference effects are often observed when light reflects from both boundary surfaces of a thin film. Figure 17.1 shows an example of reflection from two surfaces of glass plates which are separated by a layer of air having a width which changes with the position on the upper plate. Interference between the two

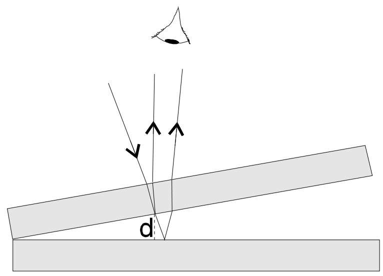

Fig. 17.1. Reflection of light from two surfaces of glass plates. The two reflected rays interfere with each other

reflected rays, as indicated in the figure is observed. At nearly normal incidence the path difference between the two rays is equal to 2d, twice the thickness of the air film at a particular point on the plate. To find the conditions of constructive or destructive interference it also has to be taken into account that both rays are reflected from media with different refractive indices. When light is reflected from a second medium having a greater refractive index than the medium of incidence it undergoes a phase change of π relative to the incidence light. When the second medium has a lower refractive index than the first there is no change in phase. Now, the condition for the constructive interference (to observe bright fringes) is

$$2d = n\lambda - \frac{1}{2}\lambda \qquad (n = 1, 2, 3, \ldots) \tag{17.3}$$

and similarly for the destructive interference (to observe dark fringes)

$$2d = \left(n + \frac{1}{2}\right)\lambda - \frac{1}{2}\lambda = n\lambda \qquad (n = 0, 1, 2, \ldots) \tag{17.4}$$

APPARATUS AND METHOD

Newton’s rings are formed as a result of the interference of light reflected from two surfaces of a thin film of air between a convex lens and a plane glass plate (figure 17.2). For monochromatic light a series of alternating dark and bright

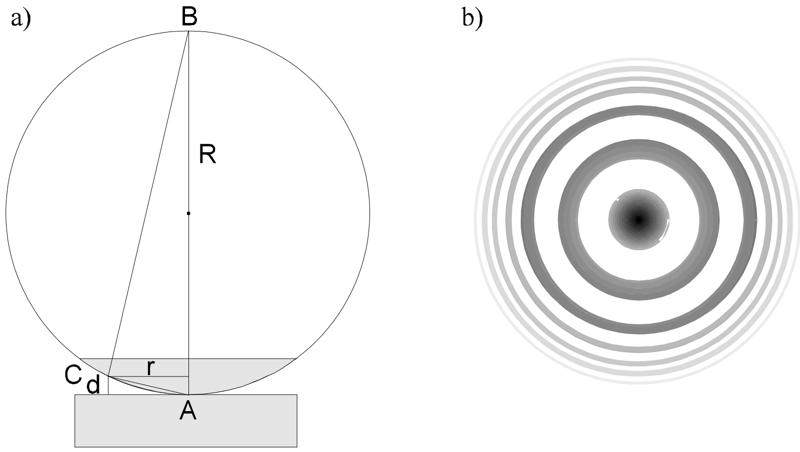

Fig. 17.2. a) A convex lens and a plane glass plate which are a source of Newton's rings of radius r. b) A picture of Newton's rings

circular fringes (rings) are observed. A ring with radius r corresponds to an air film thickness of d. From a rectangular triangle ACB the square of the radius is

$$r^2 = d(2R - d) = 2dR \tag{17.5}$$

where d has been neglected with respect to 2R, twice the radius of curvature of the lens. Introducing (17.4) for 2d it is obtained that

$$r_n^2 = R\lambda n \tag{17.6}$$

where r_n is the radius of the n dark ring.

In the experiment the Newton's rings are analyzed using a microscope with a vernier for reading the positions of the microscope table. A sodium lamp is used to provide the monochromatic light which has a wavelength of $\lambda = 589.3$ nm. The microscope table holding the convex lens and the plane plate is moved relative to the cross wires in the field of view of the microscope. The readings of the vernier are taken with the cross wires focused on the consecutive dark rings. The readings are made for the left and right side of each ring, x_n^l , x_n^r and the radius is calculated from

$$r = \frac{\left|x_n^l - x_n^r\right|}{2} \tag{17.7}$$

Equation (17.6) shows that there is a linear dependence between r_n^2 and n and the product R λ can be determined from a least-squares fit of the measurements. The radius of curvature of the lens is then given by

$$R = \frac{1}{\lambda} \frac{\sum_{n=1}^{k} n\, r_n^2}{\sum_{n=1}^{k} n^2} = \frac{6 \sum_{n=1}^{k} n\, r_n^2}{\lambda\, k(k+1)(2k+1)} \tag{17.8}$$

where k is the number of rings for which the measurements are made.

MEASUREMENTS

1. Find the radii of the first 15 dark Newton's rings. Calculate the radius of curvature of the lens using equation (17.8).
2. Draw a graph of r_n^2 against n and also for comparison, in the same figure a straight line with a slope λR going through the origin of the axis.

ANALYSIS OF ERRORS

The error in the measurement of the radius R, given as the standard deviation σ_R, is calculated from the following expression which is obtained in the least-squares fitting procedure

$$\sigma_R^2 = \frac{1}{k-2}\left(\frac{6}{\lambda^2} \frac{\sum_{n=1}^{k} r_n^4}{k(k+1)(2k+1)} - R^2 \right) \tag{17.9}$$

The error $\Delta\left(r_n^2\right)$ in the measurements of r_n^2 which determines the error bars in the plot of r_n^2 against n is equal to

$$\Delta\left(r_n^2\right) = 2\, r_n \Delta r_n \tag{17.10}$$

where Δr_n is the error in the measurements of r_n found during the taking of the data. It is assumed that the error in the wavelength λ is very small and its contribution to σ_R is neglected.

QUESTIONS

1. Explain the production of interference fringes in Young's double-slit experiment.
2. Explain how nonreflective lens coatings work?
3. Give the condition for constructive and destructive interference between reflected rays from both sides of a thin film.
4. Explain the principle of linear superposition of waves.