Experiment 18

DETERMINATION OF THE RYDBERG CONSTANT

The Rydberg constant is measured by studying the hydrogen emission spectrum in the visible region using a diffraction grating spectrometer.

INTRODUCTION

1. When a beam of light strikes an obstacle with an aperture or an edge diffraction effects in the form of fringe patterns are observed. These effects can be analyzed using Huygens' principle. It states that every point reached by an incoming wave may be considered as a source of secondary waves which propagate in all directions with the speed of the incoming wave. The resultant displacement at any point is found by adding the displacements produced by the secondary waves according to the superposition principle.

The diffraction grating is an array of a large number of parallel slits, having the same width and spacing (figure 18.1). The spacing d between the centers of

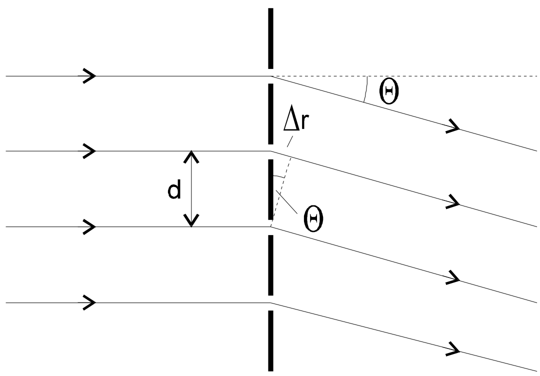

Fig. 18.1. Schematic diagram of the diffraction of a wave by a diffraction grating

adjacent slits is the grating spacing and usually is very small. The path difference Δr between two adjacent diffracted waves is $d \sin\theta$ and the maxima in their interference are observed for an angle θ which satisfies the relationship

$$d \sin\Theta = k\,\lambda \qquad (k = 0, \pm 1, \pm 2, \ldots) \tag{18.1}$$

When monochromatic light illuminates the grating a diffraction pattern containing a series of sharp lines determined by (18.1) is observed. The lines for $k = \pm 1$ are called first-order lines for $k = \pm 2$ second-order lines and so on. Diffraction gratings are used to distinguish between lines with a small difference in wavelength. The minimum wavelength difference $\Delta\lambda$ which can be observed by a grating spectrometer is given by the chromatic resolving power P

$$P = \frac{\Delta\lambda}{\lambda} = N\,k \tag{18.2}$$

which for the grating is equal to the product of the number of lines N and the line order k.

2. The emission spectrum of the hydrogen atom consists of a series of spectral lines. The wavelengths of these lines are given by the formula

$$\frac{1}{\lambda} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right) \tag{18.3}$$

R is the Rydberg constant and n is equal to m+1, m+2,... integer numbers. For $m = 2$ equation (18.3) is called the Balmer's formula and it gives the wavelengths of the Balmer series whose lines are designated by H_α (red line), H_β (blue-green line), H_γ (blue line),... with the values of their wavelengths decreasing. Other spectral series of hydrogen atom and their values of m in (18.3) are the Lyman series ($m = 1$), Paschen series ($m = 3$), and Brackett series ($m = 4$). The Lyman series is observed in the ultraviolet range of the electromagnetic spectrum.

There is a direct relation between the systematics of the above spectra and the Bohr model of the hydrogen atom. The total energy of the electron in the orbit with quantum number n is equal to

$$E_n = -\frac{m_e e^4}{8\,\varepsilon_0^2 h^2}\frac{1}{n^2} \tag{18.4}$$

The energy is negative because the electron is bound by the proton. The energy of the atom is the smallest for $n = 1$ which corresponds to the ground state of the atom (figure 18.2). For increasing n, the energy is progressively larger. In the limit of $n = \infty$ the electron is free and the hydrogen atom is ionized.

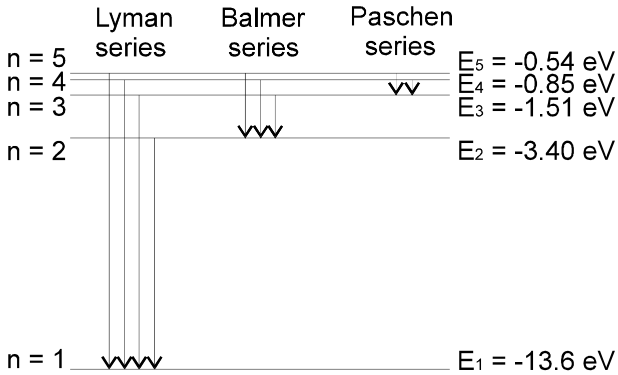

Fig. 18.2. Diagram of energy states of hydrogen atom illustrating formation of series of spectral lines. The energies of the states are given at the right of the levels

The appearance of the spectral series is explained in the Bohr model on the basis of transitions between different energy levels (figure 18.2). For the Lyman series the final state is the ground state with n = 1, for the Balmer series it is that with n = 2, and so on. Now knowing that in a transition between two energy levels m, n a line with a wavelength λ determined by

$$\frac{hc}{\lambda} = E_m - E_n \tag{18.5}$$

is emitted, it is found using (18.4) that the Rydberg constant is equal to

$$R = \frac{m_e e^4}{8\,\varepsilon_0^2 h^3 c} \tag{18.6}$$

It is expressed in terms of the fundamental physical constants, the mass of an electron, m, the charge of an electron, e, Planck's constant, h, the speed of light, c and the permittivity of vacuum, ε_0.

APPARATUS AND METHOD

The diffraction grating spectrometer is shown in figure 18.3. It consists of a source of light A, a collimator C, an optical diffraction grating G and a telescope T. The grating is adjusted to ensure that the slits are vertical and its face is perpendicular to the axis of the collimator. The collimator provides a parallel beam of light incident on the grating. The telescope is rotated about the axis of the

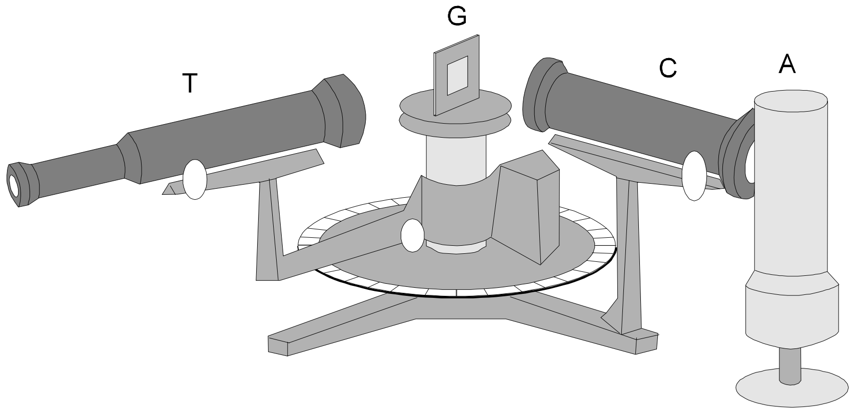

Fig. 18.3. Diffraction grating spectrometer used in the measurements of the Rydberg constant. A is a source of light, C is a collimator, G is a diffraction grating and T is a telescope

spectrometer to analyze the diffracted spectrum of the incident beam. The position of the telescope is read on the spectrometer angular scale using a vernier when the telescope is focused on a particular diffraction line. The angle of diffraction θ for each order is calculated from the positions of the telescope right and left, θ_r and θ_l, of the zero

$$\Theta = \frac{1}{2}\left| \Theta_r - \Theta_l \right| \qquad (18.7)$$

In the experiment the grating spacing d is found using a sodium lamp with the wavelength $\lambda = 589.3$ nm. The measurements are performed for all orders observed in the diffraction image and d is calculated using equation (18.1).

The Rydberg constant is determined in measurements of the diffraction angles of H_α, H_β and H_γ lines using a hydrogen discharge source. The wavelengths λ of the above lines are calculated from equation (18.1) using the grating spacing d obtained. The Rydberg constant is calculated from an expression derived from Balmer's formula

$$R = \frac{4\,n^2}{n^2 - 4}\,\frac{1}{\lambda} \qquad (18.8)$$

Here $n = 3$ for the H_α line, $n = 4$ for the H_β line and $n = 5$ for the H_γ line of hydrogen.

MEASUREMENTS

1. Find the grating spacing using the sodium lamp as a source of light. Take the measurements for all orders of diffraction observed and calculate the mean value of d.
2. Find the Rydberg constant using a discharge lamp as a source of hydrogen lines. Take the measurements for all the orders of diffraction observed and calculate the mean values of wavelengths λ. Then calculate R from (18.8) and find its mean value.
3. Calculate the Rydberg constant from equation (18.6) to compare with the value obtained experimentally. In the calculations take the values of the physical constants from table 1.

ANALYSIS OF ERRORS

The error Δd in the measurement of the grating spacing d is taken to be equal to

$$\Delta d = \left| d_m - \bar{d} \right| \tag{18.9}$$

where $\bar{d}$ is the mean value of d and d_m is the obtained value of d which deviates most from the mean value. In the same way the error ΔR in the measurement of the Rydberg constant is taken as

$$\Delta R = \left| R_m - \bar{R} \right| \tag{18.10}$$

where $\bar{R}$ is the mean value of R and R_m is the result of the measurement that deviates most.

QUESTIONS

1. Give the conditions for constructive and destructive interference of two coherent waves and explain their application to the case of the diffraction grating.
2. White light is dispersed by a glass prism and also by a diffraction grating. Explain the differences in the observed spectra.
3. Give and explain the postulates, which Bohr made in his theory of atomic hydrogen.
4. Give examples of other systems apart from the hydrogen atom for which the Bohr model can be applied.